Graph Algorithms

M269 Unit 5

Phil Molyneux

13 March 2021

M269 Graph Algorithms

Agenda & Aims

- Welcome and introductions
- Session on M269 Unit 5 *Optimisation*
- Graph definitions and representations
- Python: List comprehensions, Named Tuples
- Topological Sort for directed acyclic graphs
- Dijkstra's Shortest Path Algorithm
- Prim's Minimum Spanning Tree Algorithm
- Dynamic Programming
- Implementations in Structured English, Python and Haskell (Optional)
- *Note* there is more material here than we can cover — some is for optional interest
- Slides/Notes are at
  pmolyneux.co.uk/OU/M269/M269TutorialNotes/M269TutorialGraphs/
- **Recording** Meeting › Record Meeting. . . ✔

M269 Tutorial

Introductions — Me

- *Name* Phil Molyneux
- *Background* Physics & Maths, Operational Research, Computer Science
- *First programming languages* Fortran, BASIC, Pascal
- *Favourite Software*
  - Haskell — pure functional programming language
  - Text editors TextMate, Sublime Text — previously Emacs
  - Word processing in LaTeX — all these slides and notes
  - Mac OS X
- *Learning style* — I read the manual before using the software

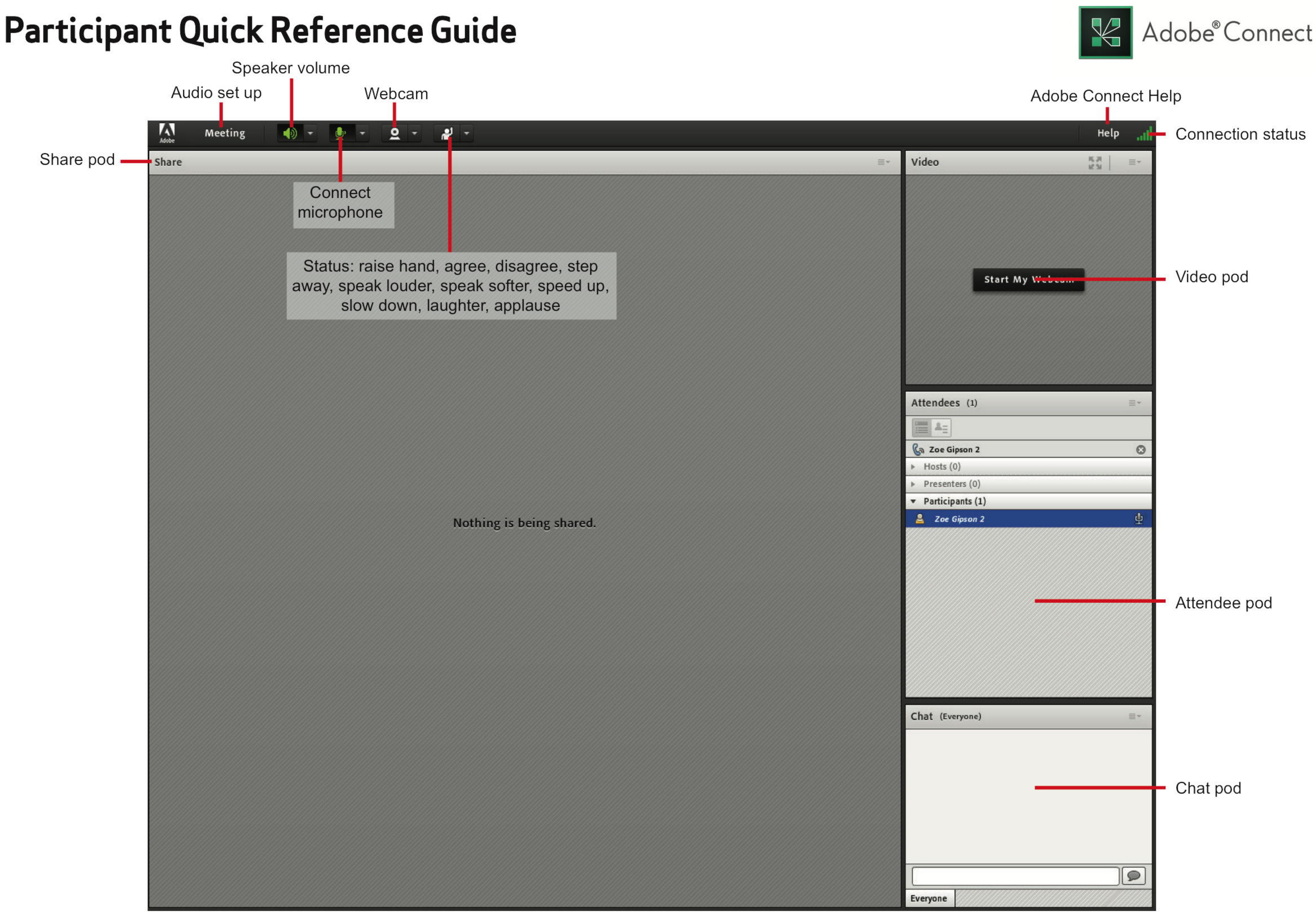
M269 Tutorial

Introductions — You

- *Name* ?
- *New topics last month* ?
- *M269 Unit 5 Graph Algorithm topics you want covered* ?
- *Learning style* ?
- *Other OU courses* ?
- *Anything else* ?
- *Adobe Connect* — if you or I get cut off, wait till we reconnect (or send you an email)

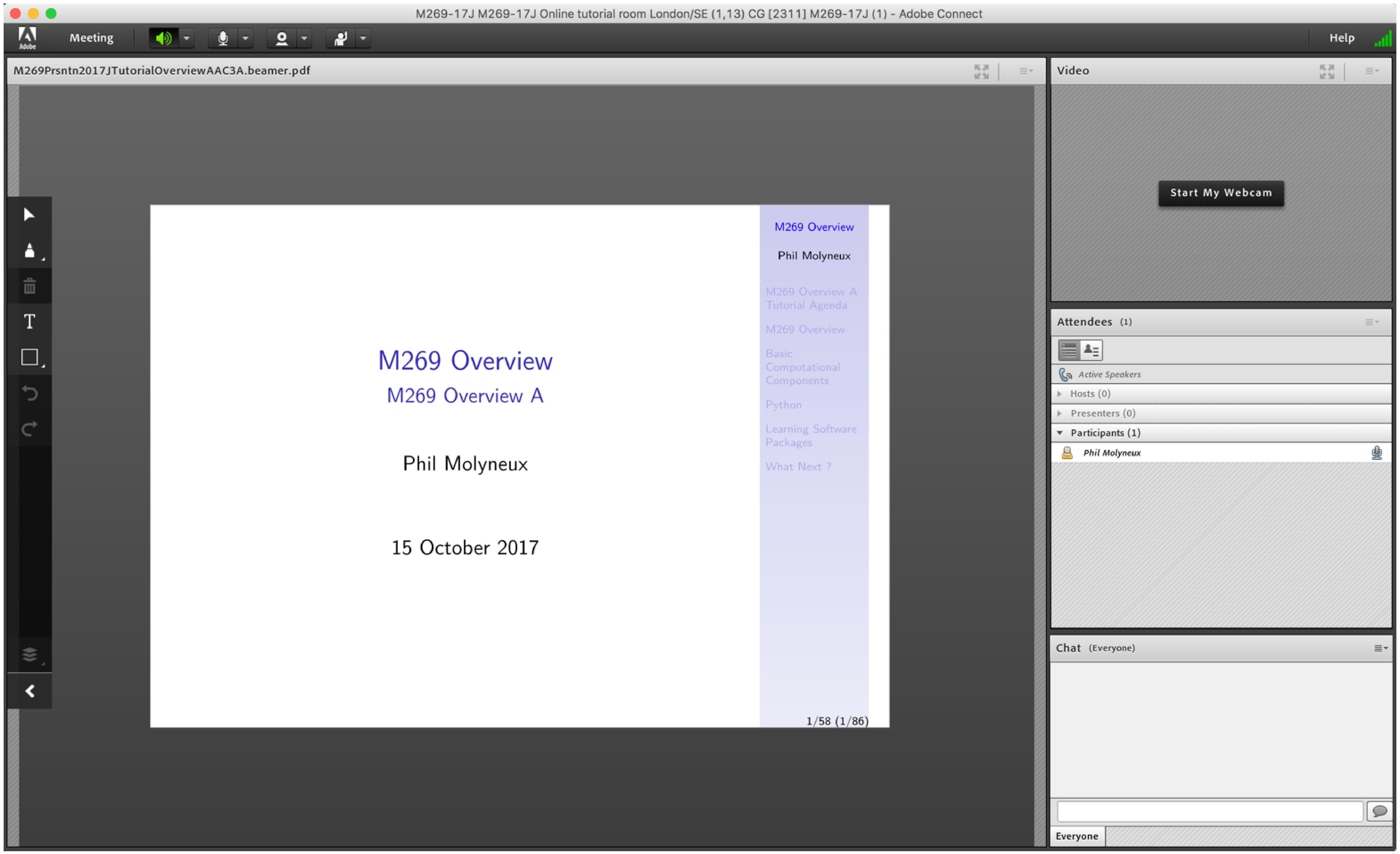
Adobe Connect

Interface — Student Quick Reference



Adobe Connect

Interface — Student View



Adobe Connect

Settings

- **Everybody: Audio Settings** Meeting › Audio Setup Wizard. . .
- **Audio** Menu bar › Audio › Microphone rights for Participants ✔
- Do not *Enable single speaker mode*
- **Drawing Tools** Share pod menu bar › Draw (1 slide/screen)
- Share pod menu bar › Menu icon › Enable Participants to draw ✔ gray
- Meeting › Preferences › Whiteboard › Enable Participants to draw ✔
- Cancel hand tool . . . Do not enable green pointer. . .
- Meeting › Preferences › Attendees Pod ✘ *Raise Hand notification*
- Meeting › Preferences › Display Name Display First & Last Name
- **Cursor** Meeting › Preferences › General tab › Host Cursors › Show to all attendees ✔ (default *Off*)
- Meeting › Preferences › Screen Share › Cursor › Show Application Cursor
- **Webcam** Menu bar › Webcam › Enable Webcam for Participants ✔
- **Recording** Meeting › Record Meeting. . . ✔

Adobe Connect
Access

- **Tutor Access**

  TutorHome › M269 Website › Tutorials

  Cluster Tutorials › M269 Online tutorial room

  Tutor Groups › M269 Online tutor group room

  Module-wide Tutorials › M269 Online module-wide room
- **Attendance**

  TutorHome › Students › View your tutorial timetables
- **Beamer Slide Scaling** 440% (422 x 563 mm)
- **Clear Everyone's Status**

  Attendee Pod › Menu › Clear Everyone's Status
- **Grant Access** and send link via email

  Meeting › Manage Access & Entry › Invite Participants. . .
- **Presenter Only Area**

  Meeting › Enable/Disable Presenter Only Area

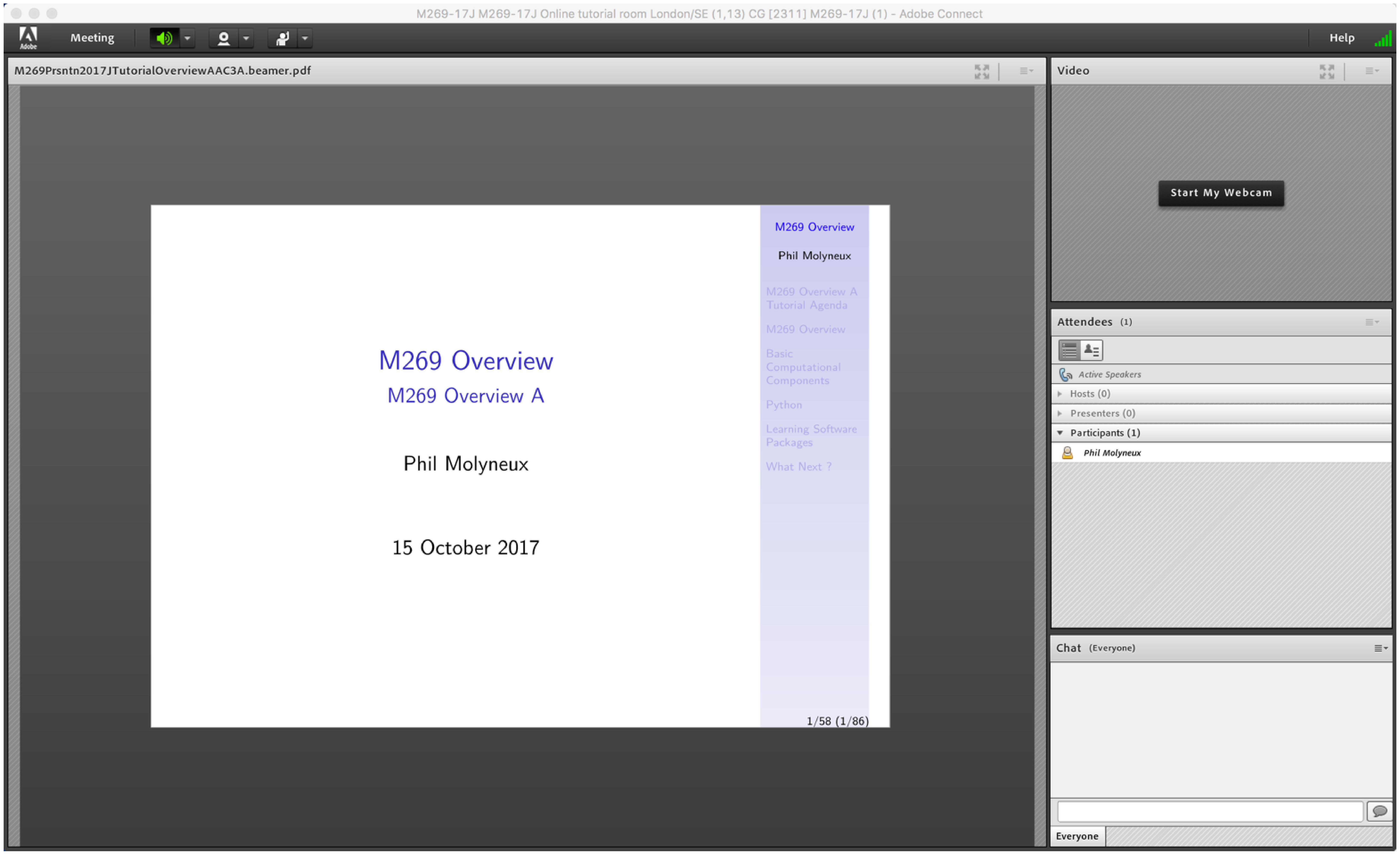
Adobe Connect

Keystroke Shortcuts

- Keyboard shortcuts in Adobe Connect
- **Toggle Mic** ⌘ + M (Mac), Ctrl + M (Win) (On/Disconnect)
- **Toggle Raise-Hand status** ⌘ + E
- **Close dialog box** ⎋ (Mac), Esc (Win)
- **End meeting** ⌘ + \

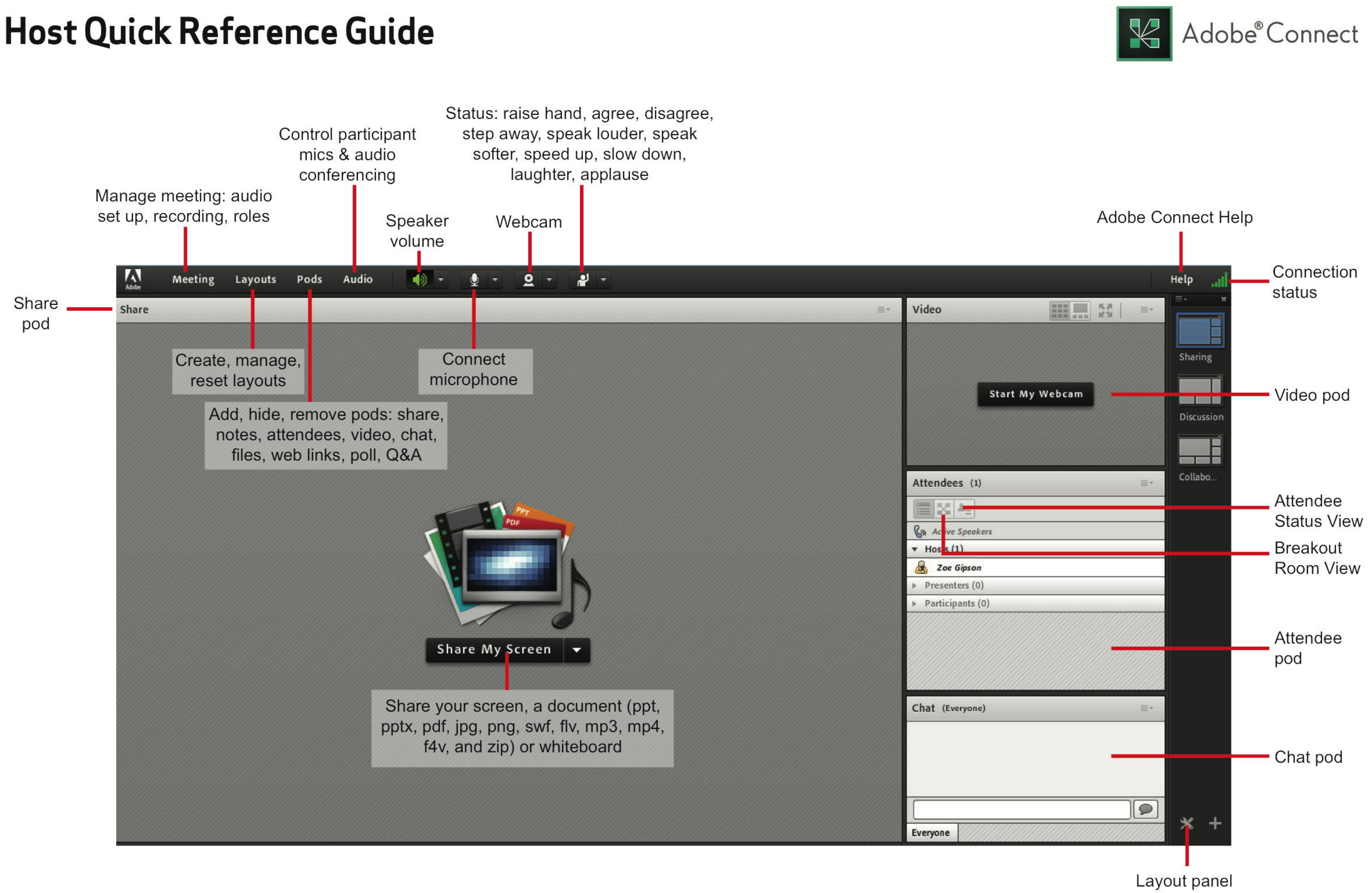
Adobe Connect Interface

Student View (default)



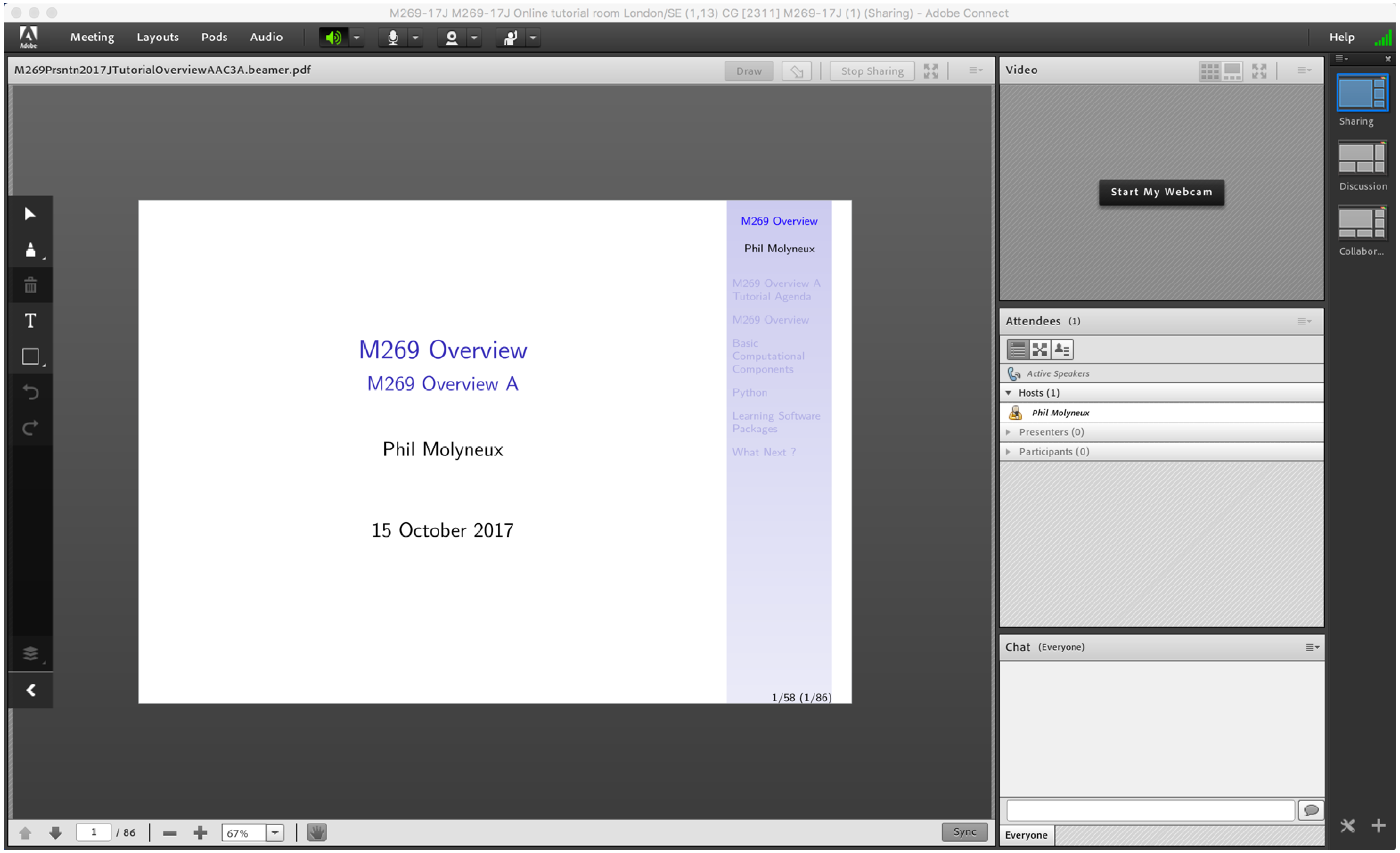
Adobe Connect Interface

Tutor View



Adobe Connect Interface

Tutor View



Adobe Connect Interface

Sharing Screen & Applications

- Share My Screen ❯ Application tab ❯ Terminal for Terminal
- Share menu ❯ Change View ❯ Zoom in for mismatch of screen size/resolution (Participants)
- (Presenter) Change to 75% and back to 100% (solves participants with smaller screen image overlap)
- Leave the application on the original display
- Beware blued hatched rectangles — from other (hidden) windows or contextual menus
- Presenter screen pointer affects viewer display — beware of moving the pointer away from the application
- First time: System Preferences ❯ Security & Privacy ❯ Privacy ❯ Accessibility

Adobe Connect

Ending a Meeting

- *Notes for the tutor only*
- **Student:** Meeting 〉 Exit Adobe Connect
- **Tutor:**
- **Recording** Meeting 〉 Stop Recording ✔
- **Remove Participants** Meeting 〉 End Meeting. . . ✔
  - Dialog box allows for message with default message:
  - *The host has ended this meeting. Thank you for attending.*
- **Recording availability** *In course Web site for joining the room, click on the eye icon in the list of recordings under your recording* — edit description and name
- **Meeting Information** Meeting 〉 Manage Meeting Information — can access a range of information in Web page.
- **Attendance Report** see course Web site for joining room

Adobe Connect

Invite Attendees

- **Provide Meeting URL** Menu › Meeting › Manage Access & Entry › Invite Participants...
- **Allow Access without Dialog** Menu › Meeting › Manage Meeting Information provides new browser window with *Meeting Information* Tab bar › Edit Information
- Check *Anyone who has the URL for the meeting can enter the room*
- Default *Only registered users and accepted guests may enter the room*
- **Reverts to default next session but URL is fixed**
- Guests have blue icon top, registered participants have yellow icon top — same icon if URL is open
- See Start, attend, and manage Adobe Connect meetings and sessions

Adobe Connect
Layouts

- **Creating new layouts** example *Sharing* layout
- Menu › Layouts › Create New Layout. . . Create a New Layout dialog › Create a new blank layout and name it *PMolyMain*
- New layout has no Pods but does have Layouts Bar open (see Layouts menu)
- **Pods**
- Menu › Pods › Share › Add New Share and resize/position — initial name is *Share* n
- **Rename Pod** Menu › Pods › Manage Pods. . . Manage Pods › Select › Rename or Double-click & rename
- Add Video pod and resize/reposition
- Add Attendance pod and resize/reposition
- Add Chat pod — name it *PMolyChat* — and resize/reposition

Adobe Connect

Layouts

- Dimensions of **Sharing** layout (on 27-inch iMac)
  - Width of Video, Attendees, Chat column 14 cm
  - Height of Video pod 9 cm
  - Height of Attendees pod 12 cm
  - Height of Chat pod 8 cm
- **Duplicating Layouts** does *not* give new instances of the Pods and is probably not a good idea (apart from local use to avoid delay in reloading Pods)

Adobe Connect

Chat Pods

- **Format Chat text**
- Chat Pod ❯ menu icon ❯ My Chat Color
- Choices: Red, Orange, Green, Brown, Purple, Pink, Blue, Black
- Note: Color reverts to Black if you switch layouts
- Chat Pod ❯ menu icon ❯ Show Timestamps

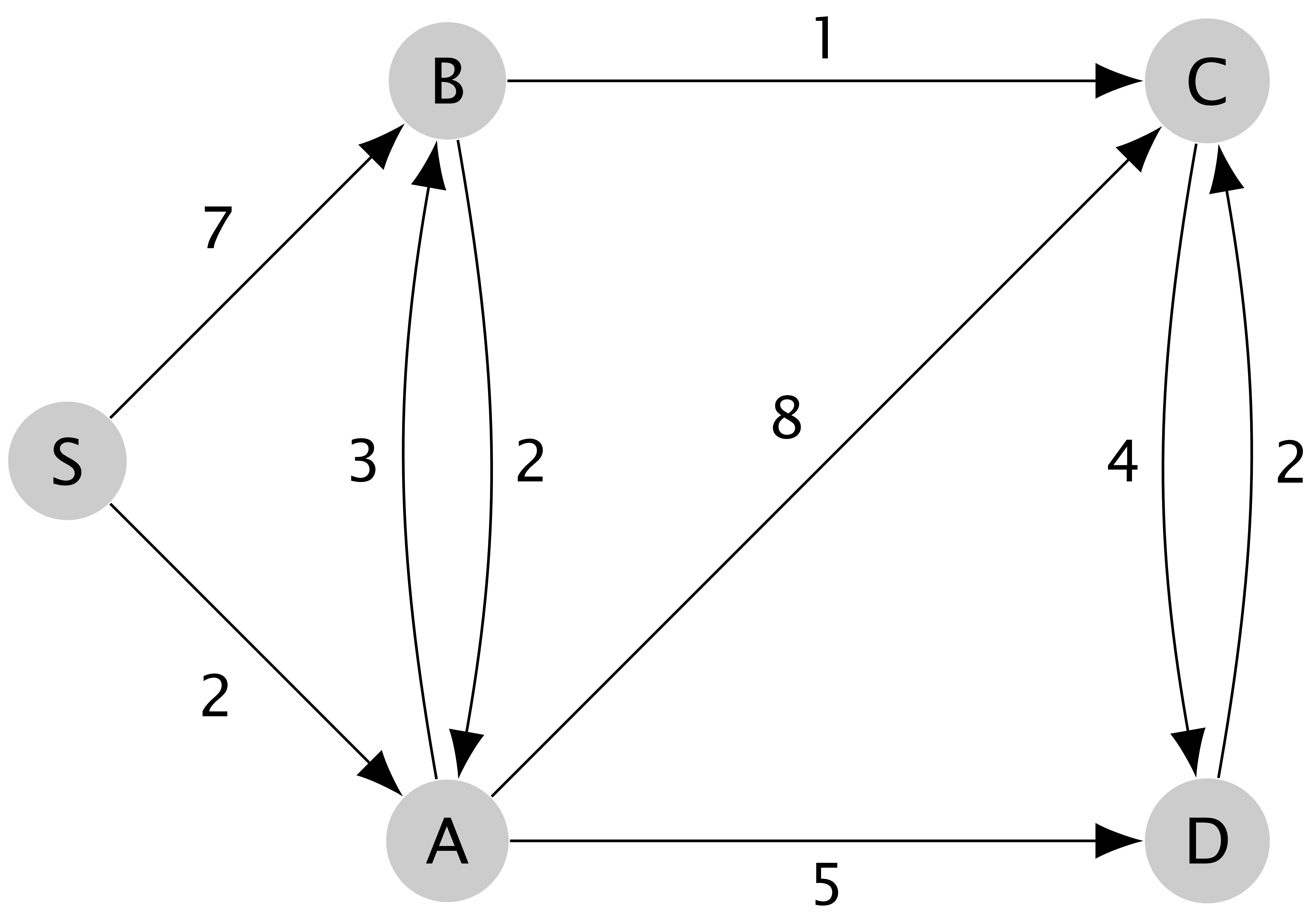
Graph Algorithms

Definitions

- A *Graph*, G, consists of a pair: a set of *vertices*, V, and a set of *edges*, E, where an edge (u, v) represents a connection between two vertices, u and v
- Equivalently, a graph is a set of objects together with a relation over that set
- Edges may have *direction* — that is, the relation is not symmetric — a graph with directed edges is called a *digraph*
- Informally, graphs are represented as diagrams (see below)
- If $G = (V, E)$ is a *weighted digraph* then there is a function $w :: E \to \mathbb{R}$ which maps edges to real numbers.
- If $e = (u, v)$ we write $w(u, v)$ for $w(e)$

Graph Algorithms

Example Digraph



Graph Algorithms

Graph Representation

- What operations do we want on graphs ?
- How can we implement a representation of graphs and the operations efficiently ?
- Common representations
  - Adjacency list — a linear structure holds every vertex together with a list of successor vertices and the weights of the successor edges.
  - Adjacency matrix — 2 dimensional array of values of dimension $|V| \times |V|$ where both coordinates u and v are vertices and the entry (u, v) is the weight of the edge (if it exists)
- Additional points:
  - A vertex may have other data: name, label with data (shortest path predecessors, distance, ...)
  - An edge may have other data: weight, status (on shortest path, minimum spanning tree, ...)

Graph Algprithms

Activity 1 Graph Operations

- In the space below give a graph operation indicating whether it is a creator, inspector or modifier and give its pre and post conditions

Go to Answer

Graph Algprithms

Answer 1 Graph Operations

- Answer 1 Graph Operations — see next slide

Go to Activity

Graph Algorithms

Graph Operations 01

- *emptyGraph* returns an empty graph
- *mkGraph* takes a list of vertices, and a list of edges and returns a graph
- *isEmptyGraph* takes a graph and returns True if and only if the graph is empty.
- *vertices* takes a graph and returns the vertices
- *edges* takes a graph and returns the edges
- *succLists* takes a graph and returns a list of pairs of vertices and lists of successor edges
- *predLists* takes a graph and returns a list of pairs of vertices and lists of predecessor edges
- *startVertices* takes a graph and returns a list of vertices with no predecessors
- *endVertices* takes a graph and returns a list of vertices with no successors

Graph Algorithms

Graph Operations 02

- *removeVertex* takes a vertex and a graph and returns a graph with the vertex removed.
- Further service functions:
  - *esRemoveV* takes a vertex and a list of edges and returns the list of edges with the vertex removed.
  - *esStartV* takes a vertex and a list of edges and returns the list of edges where the given vertex is the start of an edge
  - *esEndV* takes a vertex and a list of edges and returns the list of edges where the given vertex is the end of an edge

Graph Algorithms

Graph Representation 01

- `Adjacency matrix` Assign a unique label to each vertex and construct an $n \times n$ matrix of values in which (i, j) is x if $(i, j) \in E$ and x is its label, (i, i) is 0 and all other entries are ∞
- The adjacency matrix for the previous example digraph is:

	S	A	B	C	D
S	0	2	7	∞	∞
A	∞	0	3	8	5
B	∞	2	0	1	∞
C	∞	∞	∞	0	4
D	∞	∞	∞	2	0

Graph Algorithms

Graph Representation 02

- The explicit adjacency list or matrix representations are biased towards the procedural view of programming.
- A functional view looks for an *inductive* definition (as we had with trees)
- Functional view:
  - A graph is either the empty graph or
  - a graph extended by a new node v together with its label and with edges to those of v's successors and predecessors *that are already in the graph*
- See FGL — A Functional Graph Library and Erwig (2001)
- M269 Python examples use *adjacency lists* to represent graphs.
- The Haskell examples in these notes use a simple (but inefficient) representation to illustrate the algorithms.

Algorithm Descriptions & Implementations

Overview

- The algorithms are described in a mix of Structured English, Python and Haskell
- The Python and Haskell code does not use any advanced features but may use some features not mentioned in M269
- In Python the code may use:
  - List comprehensions (tutorial), List comprehensions (reference) — a neat way of expressing iterations over a list, came from Miranda
  - Named tuples — a *Factory Function* for tuple with named fields — *quick & dirty* objects
- The Haskell syntax is defined as it is used — novel concepts may be:
  - Algebraic Data Types — just name your user defined data type and name its elements — magic!
  - Explicit type specifications — Haskell has a very powerful type system that can help spot errors.
  - List comprehensions — as above

List Comprehensions

Python

- **List Comprehensions** provide a concise way of performing calculations over lists (or other iterables)
- Example: Square the even numbers between 0 and 9

```
Python3>>> [x ** 2 for x in range(10) if x % 2 == 0]
[0, 4, 16, 36, 64]
Python3>>> [(x,y) for x in range(4)
...               for y in range(4)
...               if x % 2 == 0
...                  and y % 3 == 0]
[(0, 0), (0, 3), (2, 0), (2, 3)]
Python3>>>
```

- In general

```
  [expr for target1 in iterable1 if cond1
        for target2 in iterable2 if cond2 ...
        for targetN in iterableN if condN ]
```

- Lots example usage in the algorithms below

List Comprehensions

Haskell

- **List Comprehensions** provide a concise way of performing calculations over lists
- Example: Square the even numbers between 0 and 9

```
GHCi> [x^2 | x <- [0..9], x `mod` 2 == 0]
[0,4,16,36,64]
GHCi>
```

- In general

```
  [expr | qual1, qual2,..., qualN]
```

- The qualifiers `qual` can be
  - Generators `pattern <- list`
  - Boolean guards — acting as filters
  - Local declarations with `let decls` for use in `expr` and later generators and boolean guards

List Comprehension Exercises

Activity 2 (a) Stop Words Filter

- Stop words are the most common words that most search engines avoid: `'a','an','the','that',...`
- Using list comprehensions, write a function `filterStopWords` that takes a list of words and filters out the stop words
- Here is the initial code

```
sentence \
 = "the␣quick␣brown␣fox␣jumps␣over␣the␣lazy␣dog"

words = sentence.split()

wordsTest \
 = (words == ['the', 'quick', 'brown'
             , 'fox', 'jumps', 'over'
             , 'the', 'lazy', 'dog'])

stopWords \
 = ['a','an','the','that']
```

Go to Answer

List Comprehension Exercises

Activity 2 (a) Stop Words Filter

```
sentence \
 = "the␣quick␣brown␣fox␣jumps␣over␣the␣lazy␣dog"

words = sentence.split()

wordsTest \
 = (words == ['the', 'quick', 'brown'
             , 'fox', 'jumps', 'over'
             , 'the', 'lazy', 'dog'])

stopWords \
 = ['a','an','the','that']
```

- Notice the Python Explicit line joining with (\<nl>) and Python Implicit line joining with ((...))
- The backslash (\) must be followed by an end of line character (<nl>)
- The ('␣') symbol represents a space (see Unicode U+2423 Open Box)

Go to Answer

List Comprehension Exercises

Activity 2 (b) Transpose Matrix

- A matrix can be represented as a list of rows of numbers
- We *transpose* a matrix by swapping columns and rows
- Here is an example

```
matrixA \
 = [[1, 2, 3, 4]
   ,[5, 6, 7 ,8]
   ,[9, 10, 11, 12]]

matATr \
 = [[1, 5, 9]
   ,[2, 6, 10]
   ,[3, 7, 11]
   ,[4, 8, 12]]
```

- Using list comprehensions, write a function `transMat`, to transpose a matrix

Go to Answer

List Comprehension Exercises

Activity 2 (c) List Pairs in Fair Order

- Write a function which takes a pair of positive integers and outputs a list of all possible pairs in those ranges
- If we do this in the simplest way we get a bias to one argument
- Here is an example of a bias to the second argument

```
yBiasLstTest \
 = (yBiasListing(5,5)
     == [(0, 0), (0, 1), (0, 2), (0, 3), (0, 4)
        , (1, 0), (1, 1), (1, 2), (1, 3), (1, 4)
        , (2, 0), (2, 1), (2, 2), (2, 3), (2, 4)
        , (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)
        , (4, 0), (4, 1), (4, 2), (4, 3), (4, 4)])
```

Go to Answer

List Comprehension Exercises

Activity 2 (c) List Pairs in Fair Order

- Rewrite the function which takes a pair of positive integers and outputs a list of all possible pairs in those ranges
- The output should treat each argument *fairly* — any initial prefix should have roughly the same number of instances of each argument
- Here is an example output

```
fairLstTest \
 = (fairListing(5,5)
     == [(0, 0)
        , (0, 1), (1, 0)
        , (0, 2), (1, 1), (2, 0)
        , (0, 3), (1, 2), (2, 1), (3, 0)
        , (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)])
```

Go to Answer

List Comprehension Exercises

Activity 2 (c) List Pairs in Fair Order

- Rewrite the function which takes a pair of positive integers and outputs a list of lists of all possible pairs in those ranges
- The output should treat each argument *fairly* — any initial prefix should have roughly the same number of instances of each argument — further, the output should be segment by each initial prefix (see example below)
- Here is an example output

```
fairLstATest \
 = (fairListingA(5,5)
     == [[(0, 0)]
        , [(0, 1), (1, 0)]
        , [(0, 2), (1, 1), (2, 0)]
        , [(0, 3), (1, 2), (2, 1), (3, 0)]
        , [(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)]])
```

Go to Answer

List Comprehension Exercises

Answer 2 (a) Stop Words Filter

- Answer 2 (a) Stop Words Filter
- *Write here:*
- Answer 2 continued on next slide

▸ Go to Activity

List Comprehension Exercises

Answer 2 (a) Stop Words Filter

- Answer 2 (a) Stop Words Filter

```
def filterStopWords(words) :
  nonStopWords \
   = [word for word in words
              if word not in stopWords]
  return nonStopWords


filterStopWordsTest \
 = filterStopWords(words) \
    == ['quick', 'brown', 'fox'
      , 'jumps', 'over', 'lazy', 'dog']
```

▸ Go to Activity

Agenda

Adobe Connect

M269 Graph Algorithms

Algorithm Descriptions & Implementations

List Comprehensions

Python Graph Representation

Haskell Graph Representation

Topological Sort

Dijkstra's Algorithm

Prim's Algorithm

Dynamic Programming

M269 Library

Future Work

List Comprehension Exercises

Answer 2 (b) Transpose Matrix

- Answer 2 (b) Transpose Matrix
- *Write here:*
- Answer 2 continued on next slide

Go to Activity

List Comprehension Exercises

Answer 2 (b) Transpose Matrix

- Answer 2 (b) Transpose Matrix

```
def transMat(mat) :
  rowLen = len(mat[0])
  matTr \
   = [[row[i] for row in mat] for i in range(rowLen)]
  return matTr

transMatTestA \
 = (transMat(matrixA)
     == matATr)
```

- Note that a list comprehension is a valid expression as a target expression in a list comprehension
- The code assumes every row is of the same length

- Answer 2 continued on next slide

Go to Activity

List Comprehension Exercises

Answer 2 (b) Transpose Matrix

- Note the differences in the list comprehensions below

```
matrixA \
 = [[1, 2, 3, 4]
   ,[5, 6, 7 ,8]
   ,[9, 10, 11, 12]]
```

```
Python3>>> [[row[i] for row in matrixA]
...                  for i in range(4)]
[[1, 5, 9], [2, 6, 10], [3, 7, 11], [4, 8, 12]]
Python3>>> [row[i] for row in matrixA
...                 for i in range(4)]
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
Python3>>> [row[i] for i in range(4)
...                 for row in matrixA]
[1, 5, 9, 2, 6, 10, 3, 7, 11, 4, 8, 12]
Python3>>> [[row[i] for i in range(4)]
...                  for row in matrixA]
[[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12]]
```

Go to Activity

List Comprehension Exercises

Answer 2 (b) Transpose Matrix

- Answer 2 (b) Transpose Matrix
- The Python NumPy package provides functions for N-dimensional array objects
- For transpose see numpy.ndarray.transpose

```
Python3>>> import numpy as np
Python3>>> ar = np.array([[1,2],[3,4]])
Python3>>> ar
array([[1, 2],
       [3, 4]])
Python3>>> arT = ar.transpose()
Python3>>> arT
array([[1, 3],
       [2, 4]])
Python3>>> ar
array([[1, 2],
       [3, 4]])
Python3>>> ar.shape
(2, 2)
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order — first version
- Write here

```
yBiasLstTest \
 = (yBiasListing(5,5)
     == [(0, 0), (0, 1), (0, 2), (0, 3), (0, 4)
        , (1, 0), (1, 1), (1, 2), (1, 3), (1, 4)
        , (2, 0), (2, 1), (2, 2), (2, 3), (2, 4)
        , (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)
        , (4, 0), (4, 1), (4, 2), (4, 3), (4, 4)])
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order
- This is the *obvious* but biased version

```
def yBiasListing(xRng,yRng) :
  yBiasLst \
   = [(x,y) for x in range(xRng)
            for y in range(yRng)]
  return yBiasLst

yBiasLstTest \
 = (yBiasListing(5,5)
     == [(0, 0), (0, 1), (0, 2), (0, 3), (0, 4)
        , (1, 0), (1, 1), (1, 2), (1, 3), (1, 4)
        , (2, 0), (2, 1), (2, 2), (2, 3), (2, 4)
        , (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)
        , (4, 0), (4, 1), (4, 2), (4, 3), (4, 4)])
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order — second version
- Write here

```
fairLstTest \
 = (fairListing(5,5)
     == [(0, 0)
         , (0, 1), (1, 0)
         , (0, 2), (1, 1), (2, 0)
         , (0, 3), (1, 2), (2, 1), (3, 0)
         , (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)])
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order — second version
- This works by making the sum of the coordinates the same for each prefix

```
def fairListing(xRng,yRng) :
  fairLst \
   = [(x,d-x) for d in range(yRng)
              for x in range(d+1)]
  return fairLst

fairLstTest \
 = (fairListing(5,5)
     == [(0, 0)
        , (0, 1), (1, 0)
        , (0, 2), (1, 1), (2, 0)
        , (0, 3), (1, 2), (2, 1), (3, 0)
        , (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)]
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order — third version
- Write here

```
fairLstATest \
 = (fairListingA(5,5)
     == [[(0, 0)]
         , [(0, 1), (1, 0)]
         , [(0, 2), (1, 1), (2, 0)]
         , [(0, 3), (1, 2), (2, 1), (3, 0)]
         , [(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)]])
```

Go to Activity

List Comprehension Exercises

Answer 2 (c) List Pairs in Fair Order

- Answer 2 (c) List Pairs in Fair Order — third version
- The *inner loop* is placed into its own list comprehension

```
def fairListingA(xRng,yRng) :
  fairLstA \
   = [[(x,d-x) for x in range(d+1)]
               for d in range(yRng)]
  return fairLstA

fairLstATest \
 = (fairListingA(5,5)
     == [[(0, 0)]
        , [(0, 1), (1, 0)]
        , [(0, 2), (1, 1), (2, 0)]
        , [(0, 3), (1, 2), (2, 1), (3, 0)]
        , [(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)]])
```

▸ Go to Activity

Algorithm Descriptions & Implementations

Python & Haskell Tutorials

- Python tutorials:
  - Beginner's Python Tutorial
  - Python Programming
  - Non-Programmer's Tutorial for Python 3
  - Non-Programmer's Tutorial for Python 2.6
- Haskell Tutorials:
  - Haskell Wikibook
  - What I Wish I Knew When Learning Haskell
  - Haskell Meta-tutorial
  - Learn You a Haskell for Great Good
  - Real World Haskell

Graph Representation

Python

```python
from collections import namedtuple

Vertex = namedtuple('Vertex'
                   ,['vtxName'])

Edge = namedtuple('Edge'
                 ,['startVtx','endVtx'])
```

- This is from Python/M269TutorialGraphs2020J.py
- Reserved identifiers are shown in this color
- User defined data constructors such as Vertex and Edge are shown in that color
- Vertex is a *named tuple* with named fields — a *quick and dirty* object — recommended by Guido van Rossum
- **Health Warning:** these notes may not be totally consistent with syntax colouring.

Example Graphs

Python

```
ta = Vertex('TA')
tb = Vertex('TB')
tc = Vertex('TC')
td = Vertex('TD')
te = Vertex('TE')
tf = Vertex('TF')
tg = Vertex('TG')
th = Vertex('TH')

eg01Vs = [ta,tb,tc,td,te,tf,tg,th]

eg01Es = [(ta,tb),(tg,tb),(tg,th),(tb,tc)
         ,(tb,tf),(tf,th),(tc,td),(td,te),(te,th)]

eg01Gr = (eg01Vs, eg01Es)

eg02Es = [(ta,tb),(tb,tc),(tc,ta)] # cycles

eg02Gr = ([ta,tb,tc], eg02Es)
```

- Used ordinary tuples for edges here

Graph Service Functions

Python (1)

```python
def vertices(gr):
  return gr[0]

def edges(gr):
  return gr[1]

def esStartV(v,es):
  return [edge for edge in es if edge[0] == v]

def esEndV(v,es):
  return [edge for edge in es if edge[1] == v]

def esRemoveV(v,es):
  return [edge for edge in es
               if edge[0] != v and edge[1] != v]
```

- Choice of service function (or class methods) is a design issue — a bit of a fudge here (to avoid complexity in these notes)

Graph Service Functions

Python (2)

```
def succLists(gr):
  return [(v, esStartV(v, (edges(gr))))
          for v in vertices(gr)]

def predLists(gr):
  return [(v, esEndV(v, (edges(gr))))
          for v in vertices(gr)]

def isEmptyGraph(gr):
  return gr[0] == [] and gr[1] == []

def startVertices(gr):
  return [pLst[0] for pLst in predLists(gr)
                  if pLst[1] == []]

def endVertices(gr):
  return [sLst[0] for sLst in succLists(gr)
                  if sLst[1] == []]
```

Graph Service Functions

Python (3)

```
74 def removeVertex(v, gr):
75   vs = gr[0]
76   vs1 = vs[:]
77   if v in vs1:
78     vs1.remove(v)
79   es = gr[1]
80   es1 = esRemoveV(v,es)
81   return (vs1,es1)
```

- Note that vs1 at line 76 is a (shallow) copy of vs
- If vertices had more structure we might have to write a function to do a proper copy

Graph Representation

Haskell

```haskell
module M269TutorialGraphs2020J where
  import Data.List
  import Data.Maybe
```

1. A Haskell script starts with a module header which starts with the reserved identifier, `module` followed by the module name, `M269TutorialGraphs2020J`
2. The module name must start with an upper case letter and is the same as the file name (without its extension of *.lhs*)
3. Haskell uses *layout* (or the *off-side rule*) to determine scope of definitions, similar to Python
4. The body of the module follows the reserved identifier `where` and starts with two `import` declarations
5. These import the built-in libraries `Data.List` and `Data.Maybe`

Example Graphs

Haskell (1)

```haskell
data Vertex = TA | TB | TC | TD | TE | TF | TG | TH
              deriving (Eq, Show, Read)

type Edge = (Vertex, Vertex)

type Graph = ([Vertex],[Edge])
```

- The reserved identifier data introduces the name of a user defined type Vertex
- Following the = are the names or constructors of the elements of the type — in this case the constructors take no arguments
- The deriving part of the declaration automates the production of instances of the *Type Classes* Eq for equality and Show and Read for writing and reading.
- The reserved identifier type introduces *type synonym declarations*

Example Graphs

Haskell (2)

```haskell
eg01VS = [TA, TB, TC, TD, TE, TF, TG, TH]

eg01ES = [(TA,TB),(TG,TB),(TG,TH),(TB,TC)
         ,(TB,TF),(TF,TH),(TC,TD),(TD,TE),(TE,TH)]

eg01Gr = (eg01VS, eg01ES)

eg02VS = [TA, TB, TC]

eg02ES = [(TA,TB),(TB,TC),(TC,TA)] -- cycles

eg02Gr = (eg02VS, eg02ES)
```

Graph Service Functions

Haskell (1)

```
vertices :: Graph -> [Vertex]
vertices (vs,es) = vs

edges :: Graph -> [Edge]
edges (vs,es) = es


esStartV :: Vertex -> [Edge] -> [Edge]
esStartV v es = [(v1,v2) | (v1,v2) <- es, v1 == v]

esEndV :: Vertex -> [Edge] -> [Edge]
esEndV v es = [(v1,v2) | (v1,v2) <- es, v2 == v]

esRemoveV :: Vertex -> [Edge] -> [Edge]
esRemoveV v es = [(v1,v2) | (v1,v2) <- es,
                            v1 /= v && v2 /= v]
```

Graph Service Functions

Haskell (2)

```
succLists :: Graph -> [(Vertex,[Edge])]
succLists gr = [(v,esStartV v (edges gr))
                | v <- vertices gr]

predLists :: Graph -> [(Vertex,[Edge])]
predLists gr = [(v,esEndV v (edges gr))
                | v <- vertices gr]

isEmptyGraph :: Graph -> Bool
isEmptyGraph (vs,es) = vs == [] && es == []

startVertices :: Graph -> [Vertex]
startVertices gr = [v | (v, []) <- predLists gr]

endVertices :: Graph -> [Vertex]
endVertices gr = [v | (v, []) <- succLists gr]

removeVertex :: Vertex -> Graph -> Graph
removeVertex v (vs,es) = (delete v vs, esRemoveV v es)
```

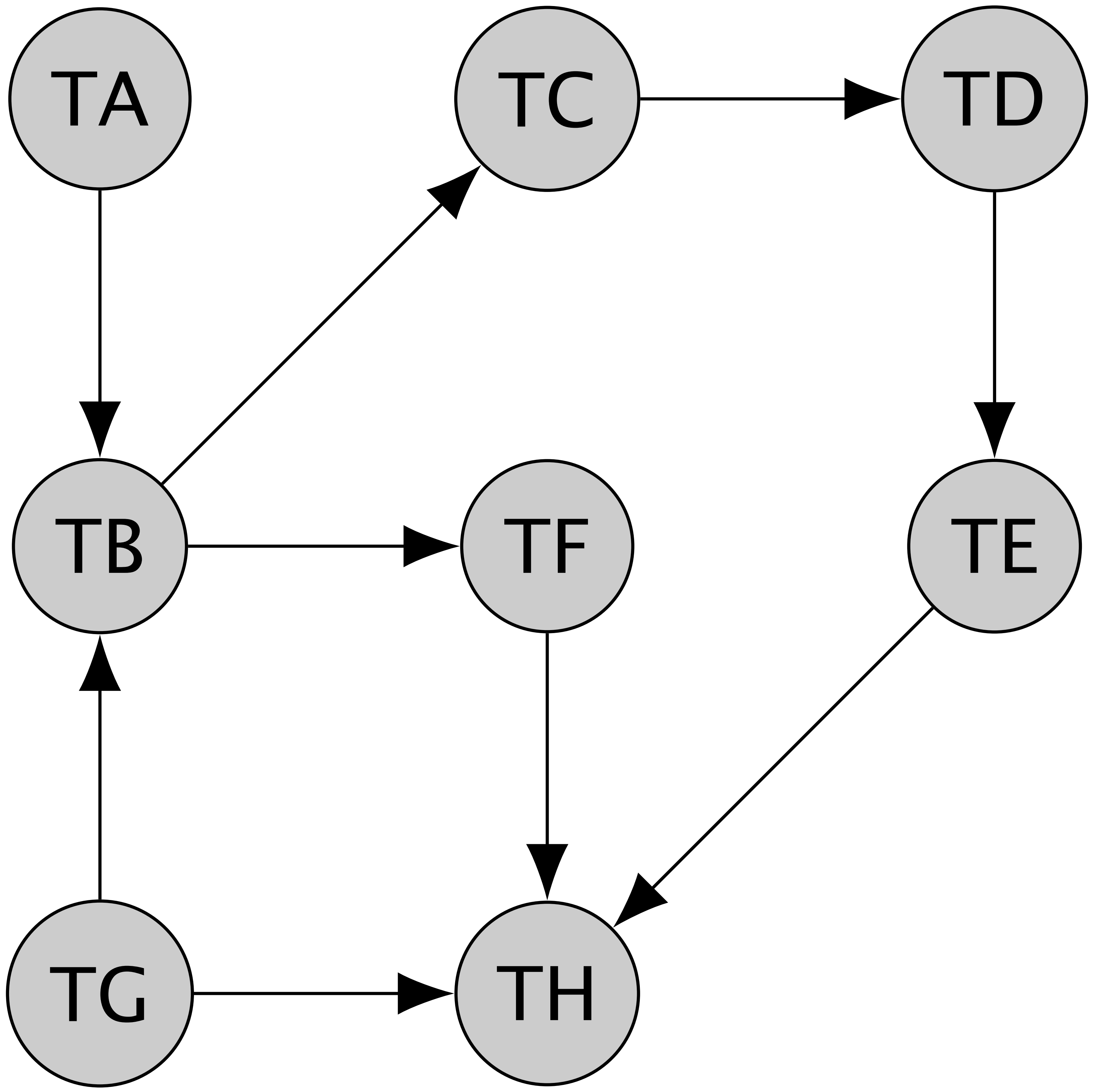
Topological Sort

Definition

- A *topological sort* of a directed acyclic graph (DAG) is a linear ordering of its vertices so that for any directed edge (u, v), u comes before v in the ordering
- See en.wikipedia.org/wiki/Topological_sorting
- A topological ordering is possible for a graph if and only if it is a DAG
- Any DAG has at least one topological ordering
- If a Hamiltonian path exists (a path visiting every node in a graph exactly once) then the graph has exactly one topological ordering

Topological Sort

Example Graph



- Find all the topological orderings on this digraph

Topological Sort

Algorithm

- `topSorts` takes a graph, `gr` and returns a list of lists of vertices (all the topological sorts of the graph)
- If the graph is empty, it returns a list containing just the empty list — *Note: not just the empty list*
- Obtain a list of all the start vertices of `gr`
- If the list of start vertices is empty, then the graph has a cycle — so raise an error and stop
- Otherwise for each start vertex, `v`
  - Join it to `ts`
  - where `ts` is one of the topological sorts of `gr` with `v` removed

Topological Sort — Algorithm

Python

```python
def topSorts(gr):
  if isEmptyGraph(gr):
    return [[]]
  elif startVertices(gr) == []:
    raise RuntimeError('Cycle in the graph')
  else:
    return [[v] + ts
            for v in startVertices(gr)
            for ts in topSorts(removeVertex(v,gr))]
```

Topological Sort — Algorithm

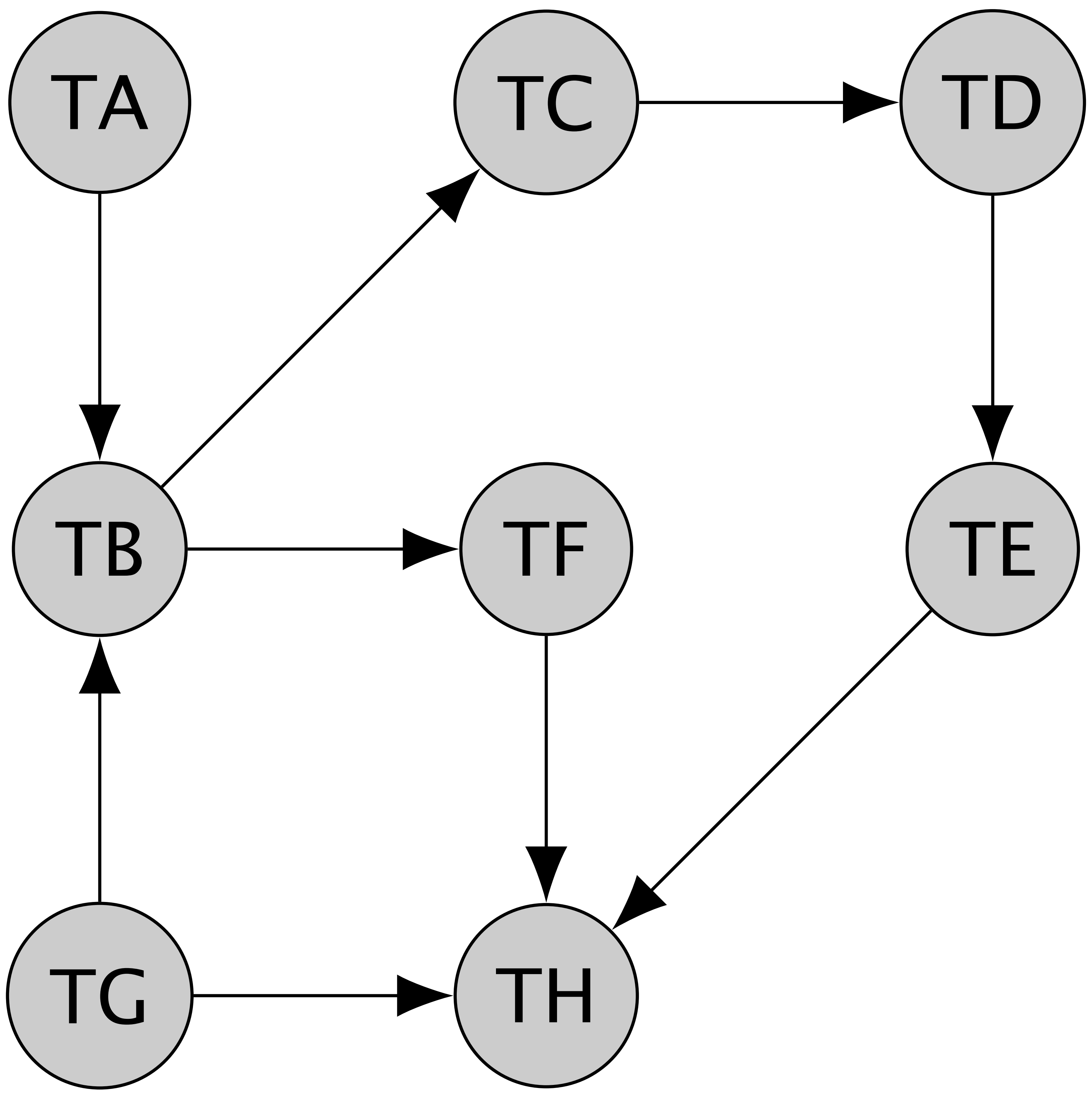
Haskell

```haskell
topSorts :: Graph -> [[Vertex]]
topSorts gr
  | isEmptyGraph gr = [[]]
  | startVertices gr == [] = error "Cycle␣in␣the␣graph"
  | otherwise
      = [v : ts | v <- startVertices gr,
                  ts <- topSorts (removeVertex v gr)]
```

Topological Sort Example 01

Activity 3 Trace Exercise

- Trace the development of the topological sort algorithm in the following graph



Go to Answer

Topological Sort Example 01

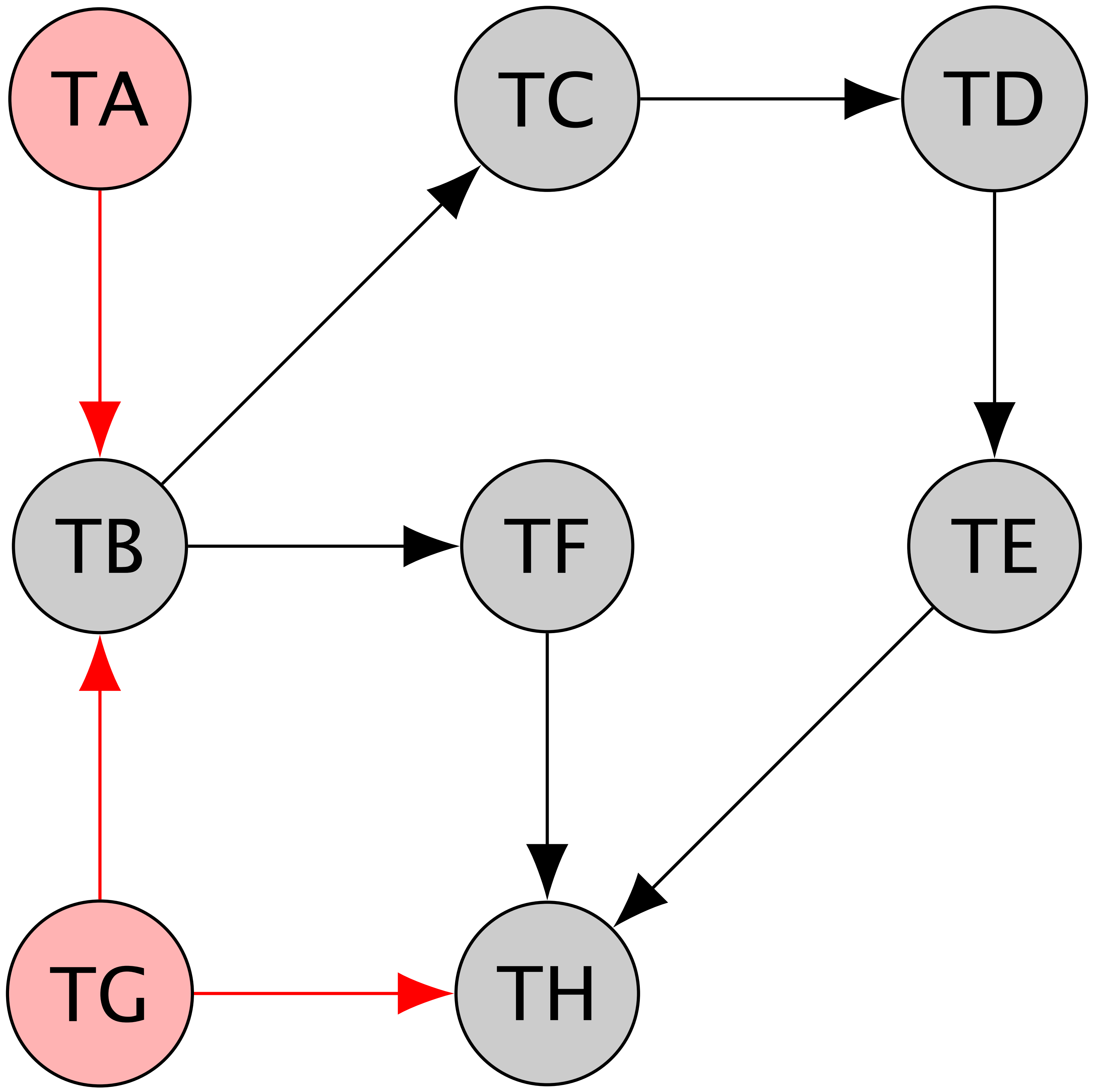
Answer 3 Trace Exercise

- Answer 3 Trace Exercise
- See the following slides

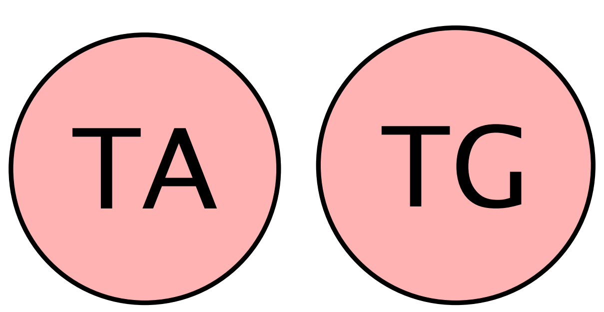
Go to Activity

Topological Sort Example 01

Step 1 Initial Graph

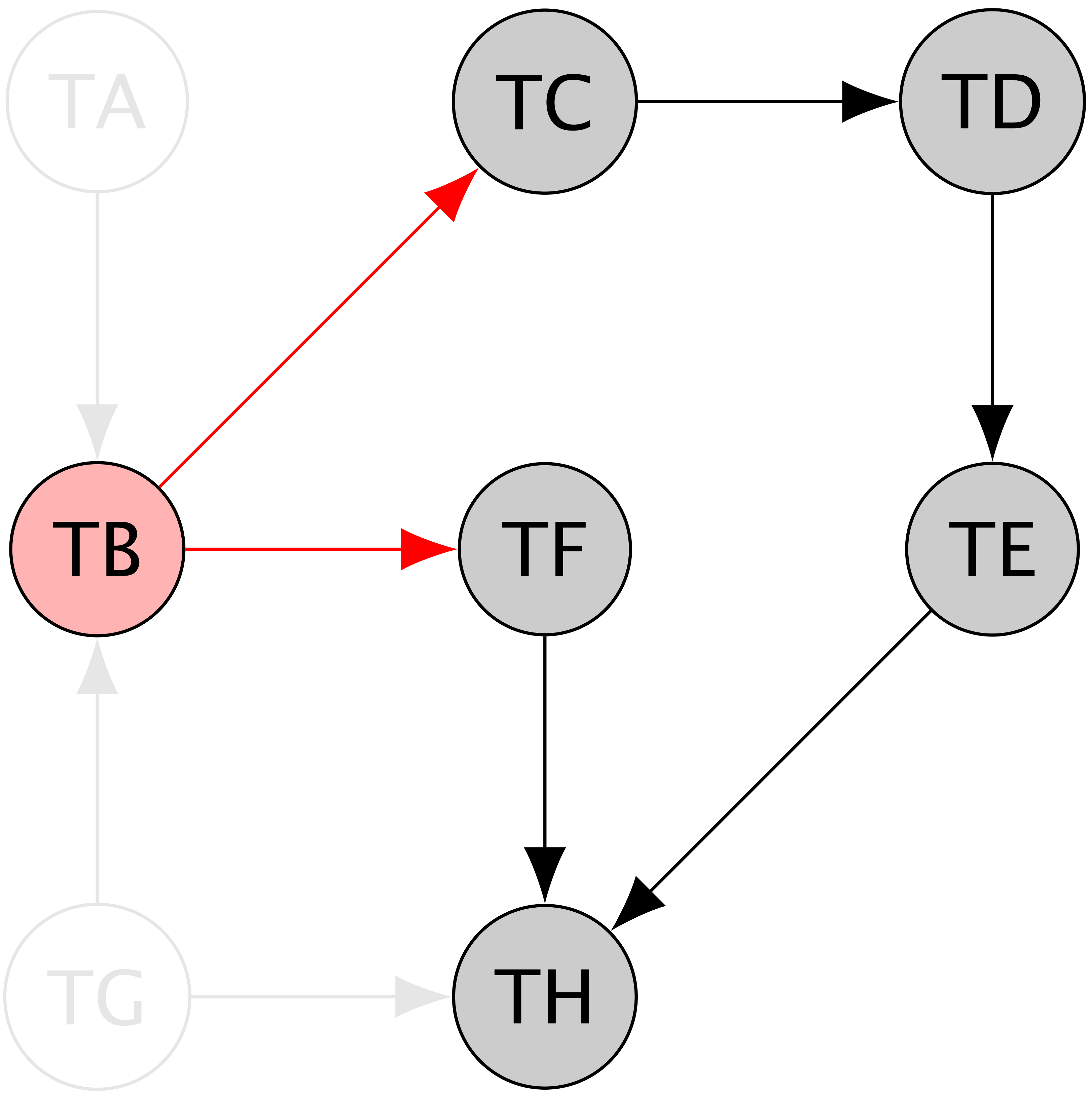


- Start vertices

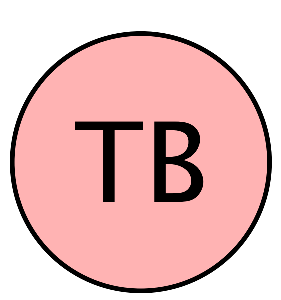


Topological Sort Example 01

Step 2 Remove Vertices TA, TG

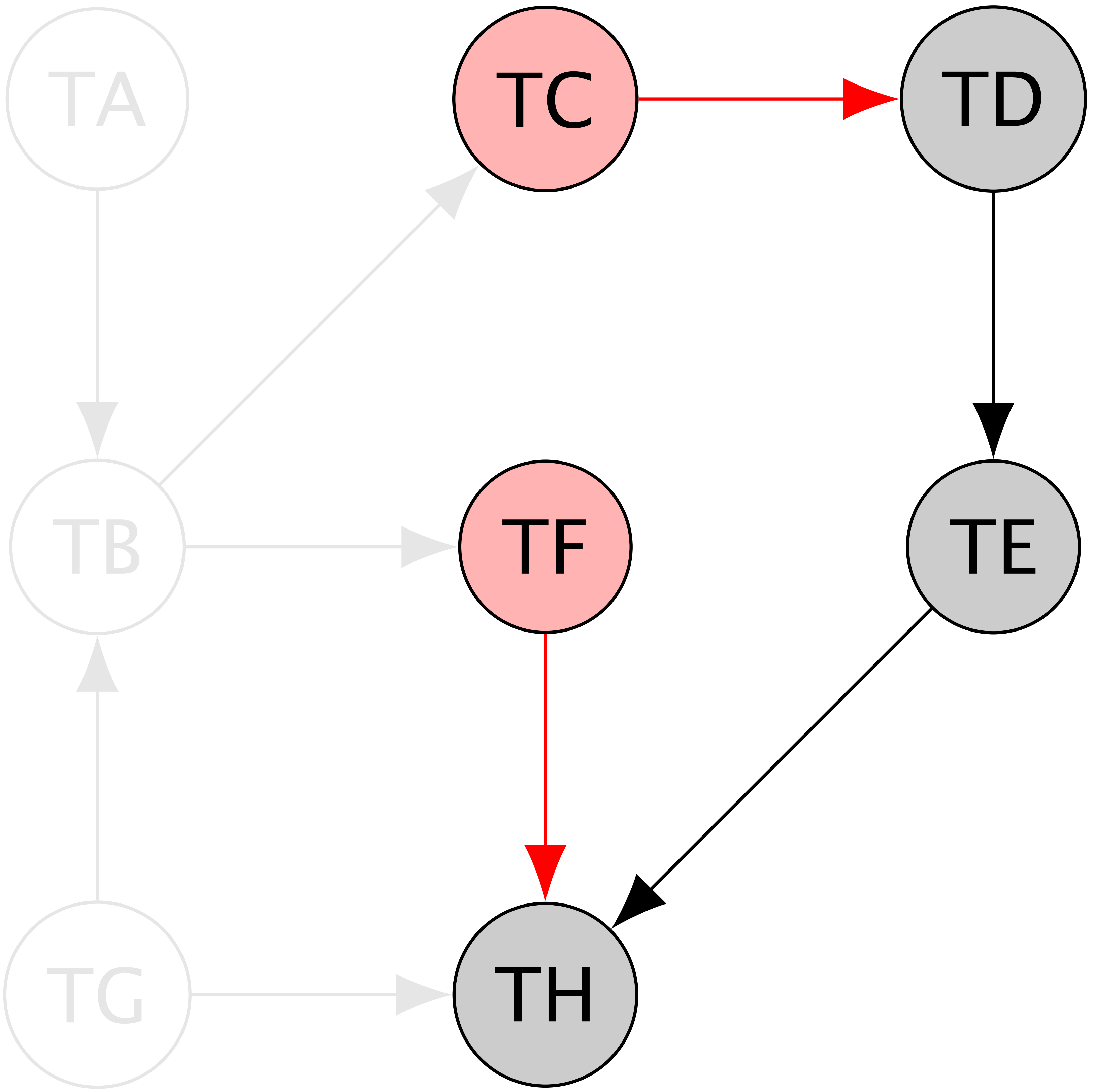


- Start vertices

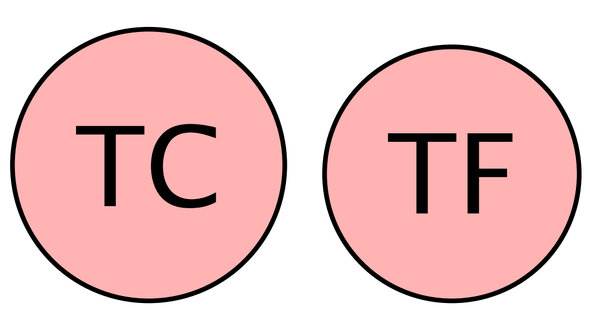


Topological Sort Example 01

Step 3 Remove Vertex TB

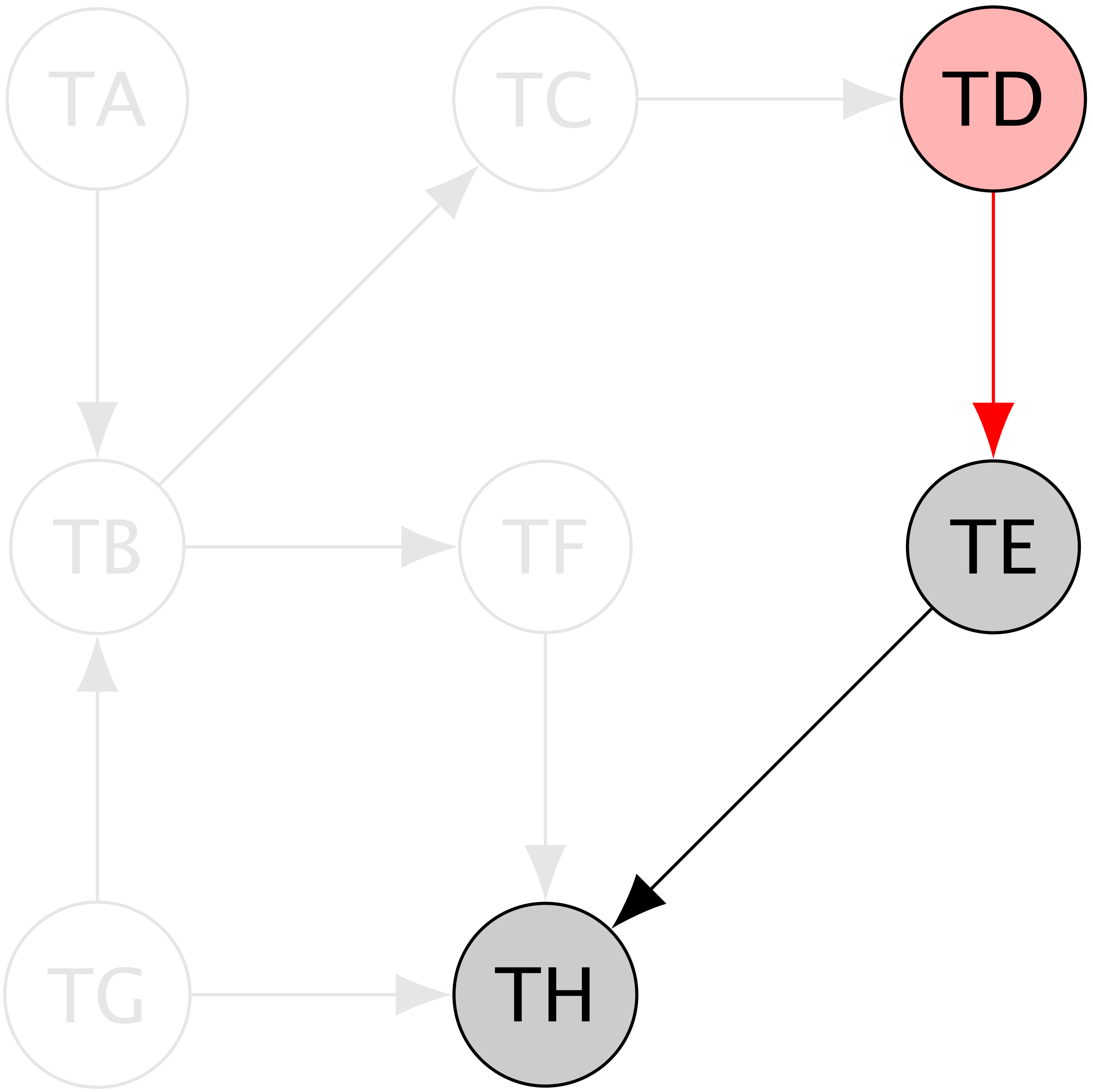


- Start vertices

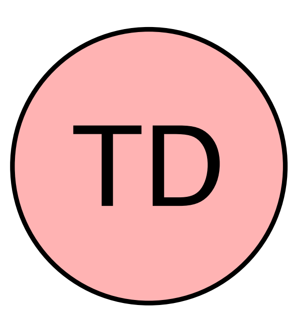


Topological Sort Example 01

Step 4 Remove Vertices TC, TF



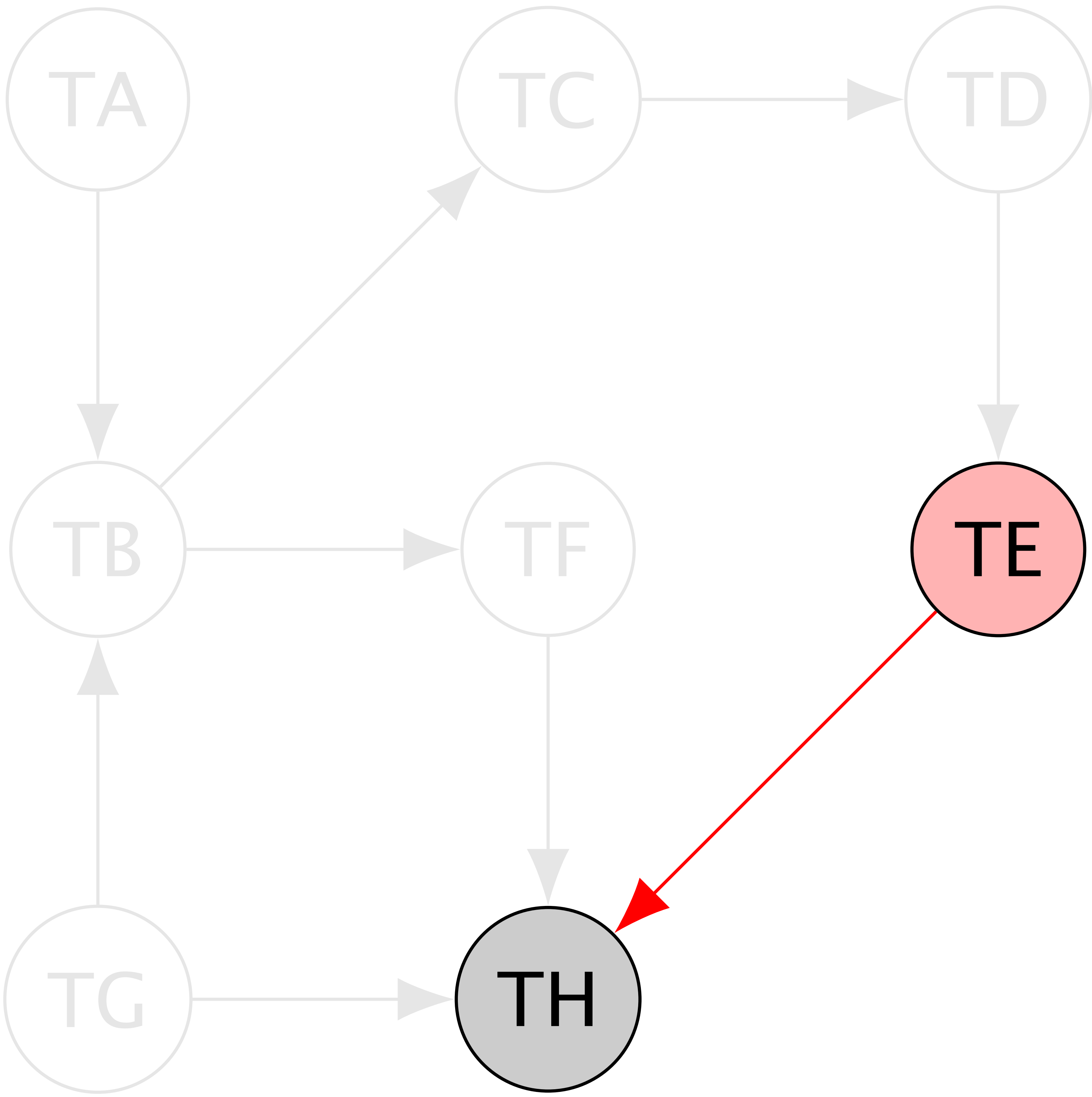
- Start vertices



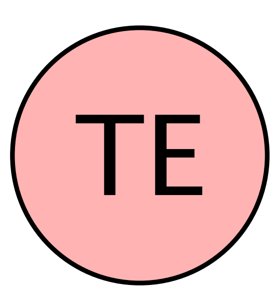
- Note: Step 4 to 6 has 4 combinations (see below)

Topological Sort Example 01

Step 5 Remove Vertex TD

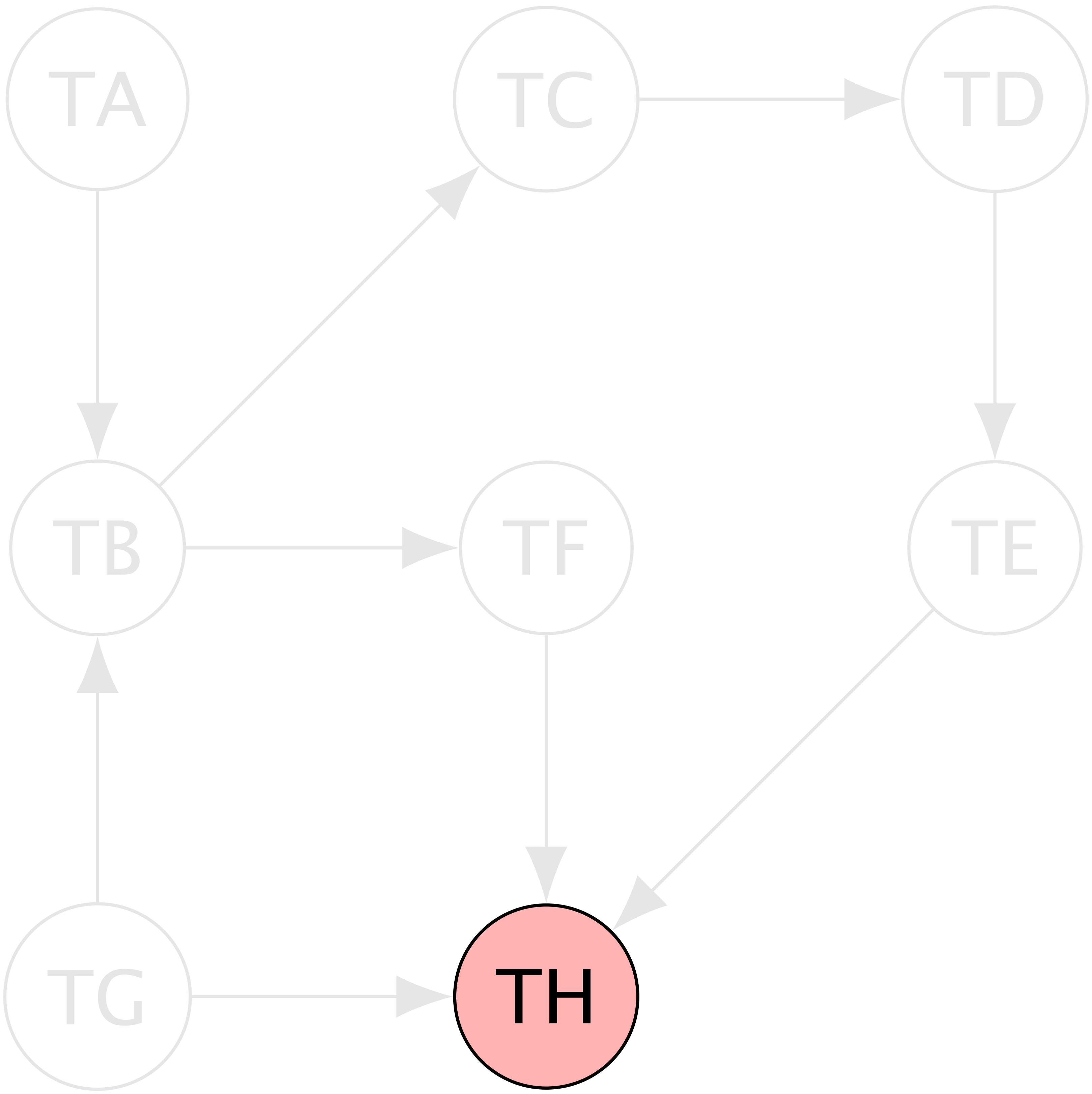


- Start vertices

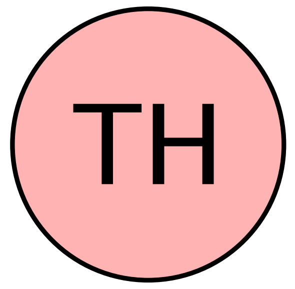


Topological Sort Example 01

Step 6 Remove Vertex TE



- Start vertices



- Step 7 would be the empty graph (not drawn)

Topological Sort — Example

Output — Python

```
topSortsEG01GrTest \
  = (topSorts(eg01Gr)
      == [[ta,tg,tb,tc,td,te,tf,th]
         ,[ta,tg,tb,tc,td,tf,te,th]
         ,[ta,tg,tb,tc,tf,td,te,th]
         ,[ta,tg,tb,tf,tc,td,te,th]
         ,[tg,ta,tb,tc,td,te,tf,th]
         ,[tg,ta,tb,tc,td,tf,te,th]
         ,[tg,ta,tb,tc,tf,td,te,th]
         ,[tg,ta,tb,tf,tc,td,te,th]])
```

- Note how the step 4 to 6 combinations get enumerated
- Note that a vertex `ta` would be displayed as `Vertex(vtxName='TA')`
- Notice the Python Explicit line joining with (`\<nl>`) and Python Implicit line joining with (`(...)`)
- The backslash (`\`) must be followed by an end of line character (`<nl>`)

Topological Sort

Output — Haskell

```
topSortsEG01GrTest
  = topSorts eg01Gr
    == [[TA,TG,TB,TC,TD,TE,TF,TH]
       ,[TA,TG,TB,TC,TD,TF,TE,TH]
       ,[TA,TG,TB,TC,TF,TD,TE,TH]
       ,[TA,TG,TB,TF,TC,TD,TE,TH]
       ,[TG,TA,TB,TC,TD,TE,TF,TH]
       ,[TG,TA,TB,TC,TD,TF,TE,TH]
       ,[TG,TA,TB,TC,TF,TD,TE,TH]
       ,[TG,TA,TB,TF,TC,TD,TE,TH]]
```

- Note how the step 4 to 6 combinations get enumerated

Dijkstra's Algorithm

Sources

- From https://www.cse.ust.hk/~dekai/271/ (Lecture 10)
- Cormen (2009, chapter 24) — Cormen (2009, page 648) has a footnote explaining the origin of the term *relaxation*
- Sedgewick and Wayne (2011)
- Miller and Ranum (2011, section 7.8)
- *A Functional Graph Library* http://web.engr.oregonstate.edu/~erwig/fgl/
- Rabhi and Lapalme (1999, chapter 7)

Dijkstra's Algorithm

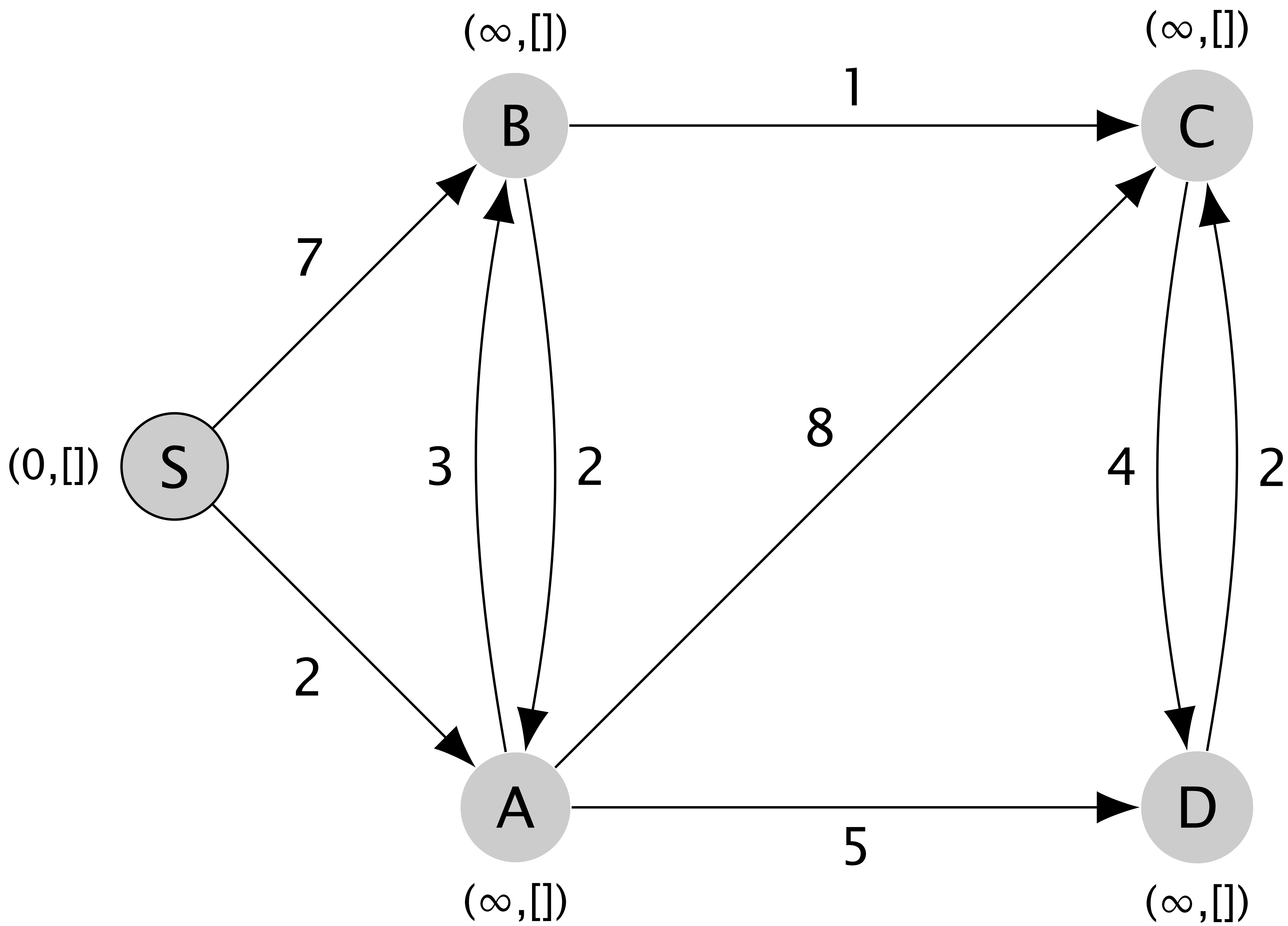
Structured English

```
dijkstra(gr,weight,s)
  for u in vertices(gr)
    dist(u) = Infinity
    label(u) = Temp
  dist(s) = 0
  pred(s) = None
  q = makePriorityQ(vertices(gr))

  while not isEmptyPQ(q)
    u = extractMinPQ(q)
    for v in adj(gr,u)
      if (label(v) == Temp
          and dist(u) + weight(edge(u,v)) < dist(v))
        dist(v) = dist(u) + weight(edge(u,v))
        q = decreaseKeyPQ(q,v,dist(v))
        pred(v) = u
    label(u) = Permanent
```

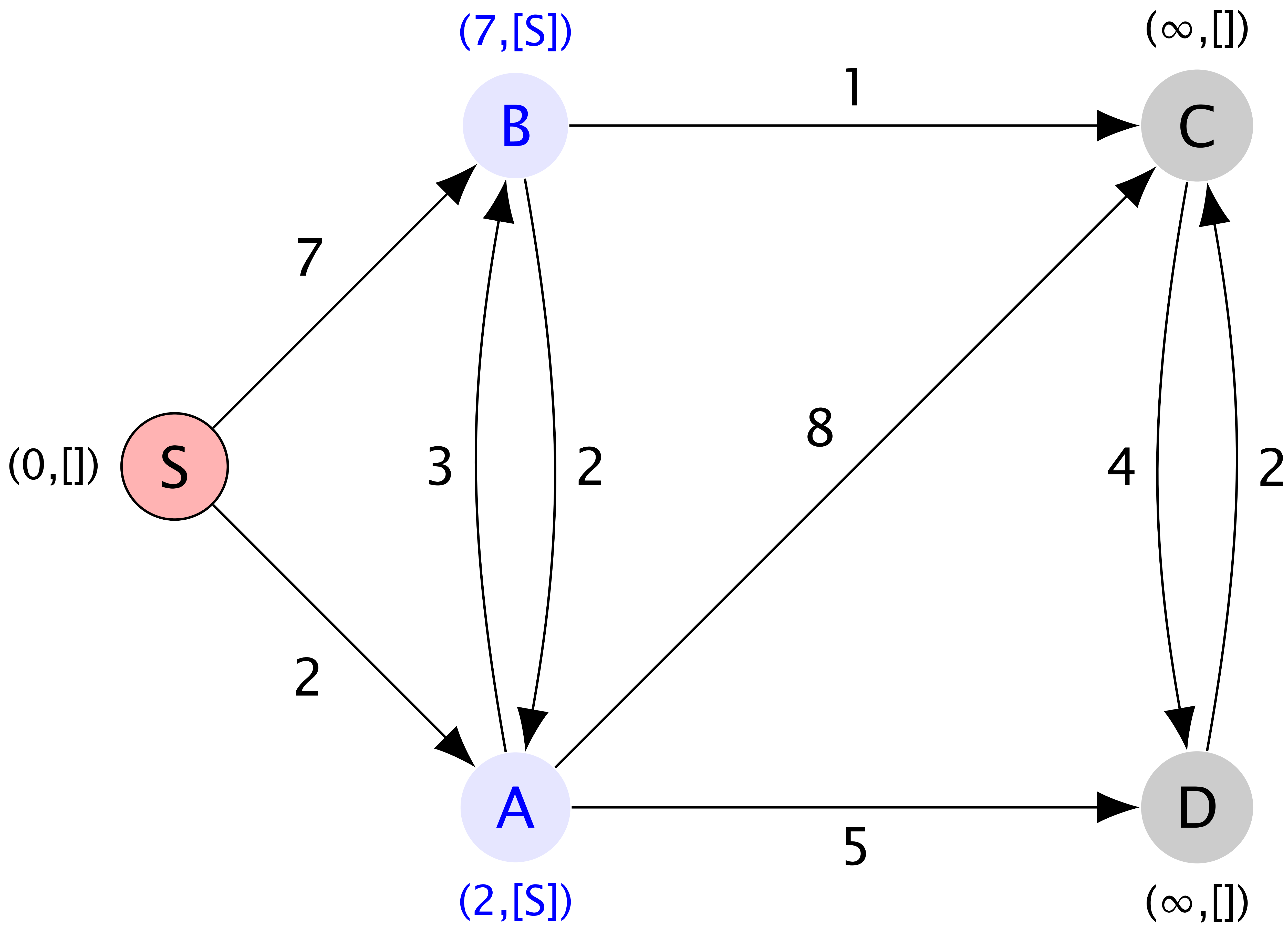
Dijkstra’s Algorithm Example 01

Step 0 Initialisation



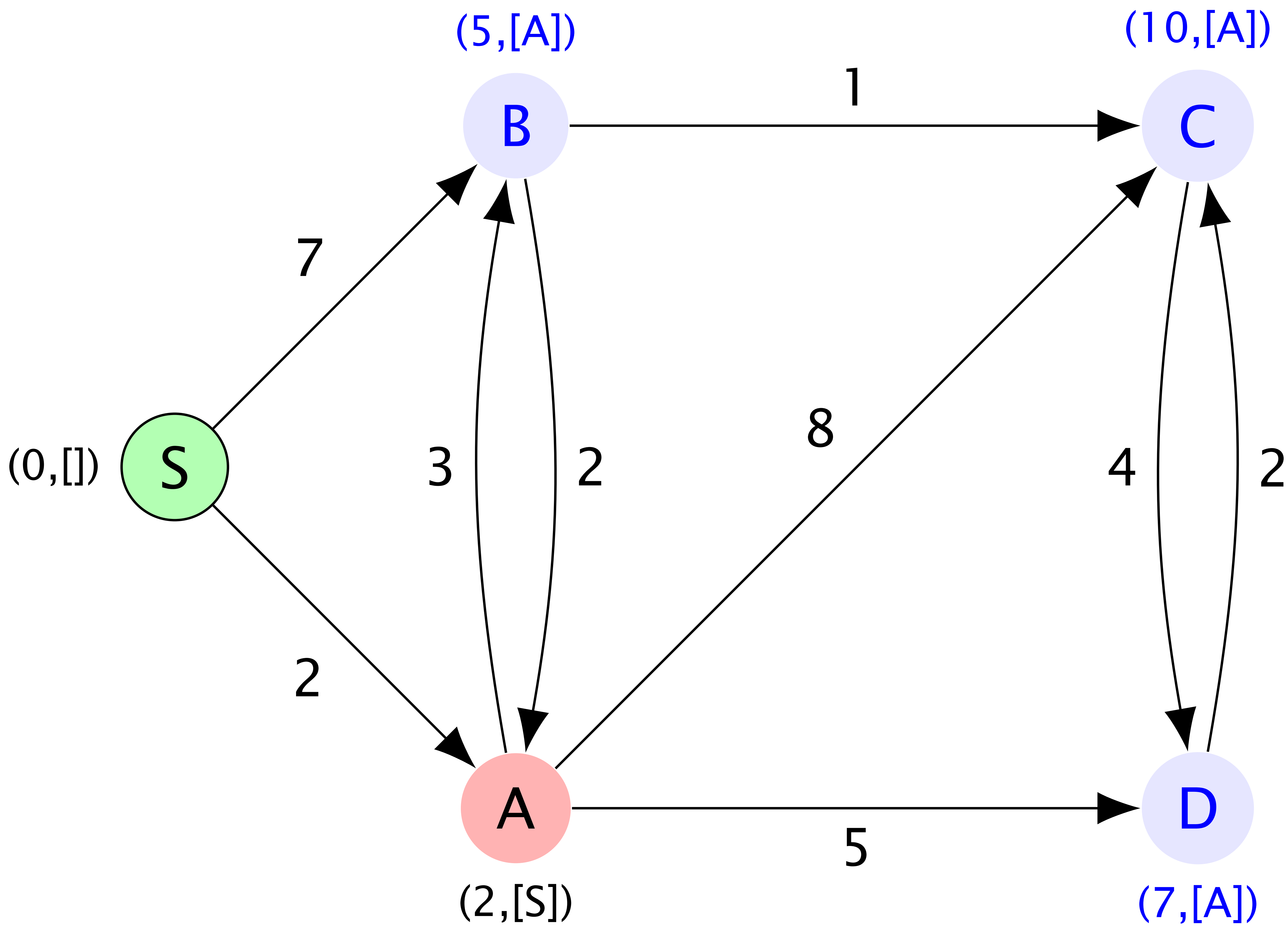
Dijkstra's Algorithm Example

Step 1 Process S



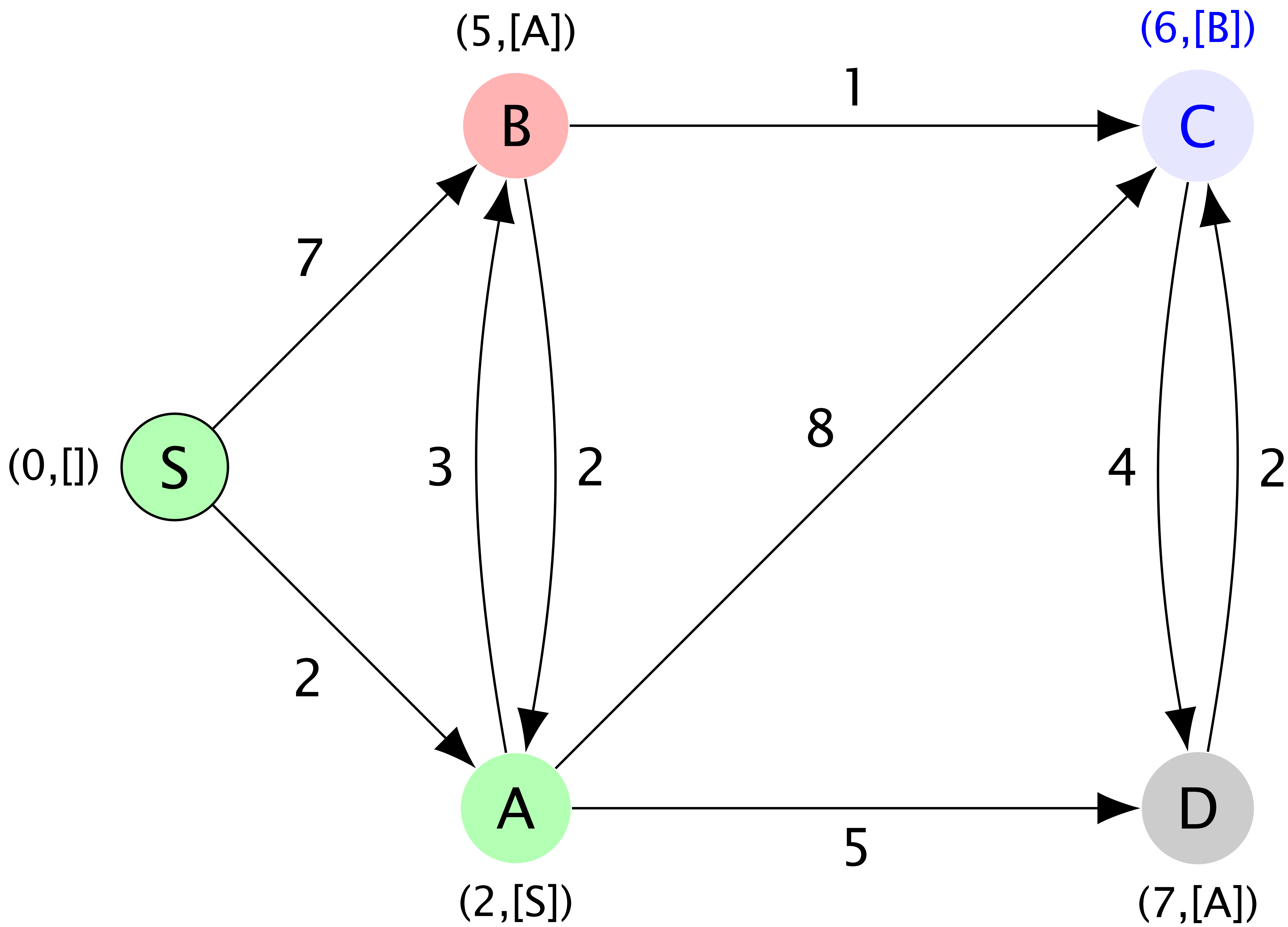
Dijkstra's Algorithm Example

Step 2 Process A



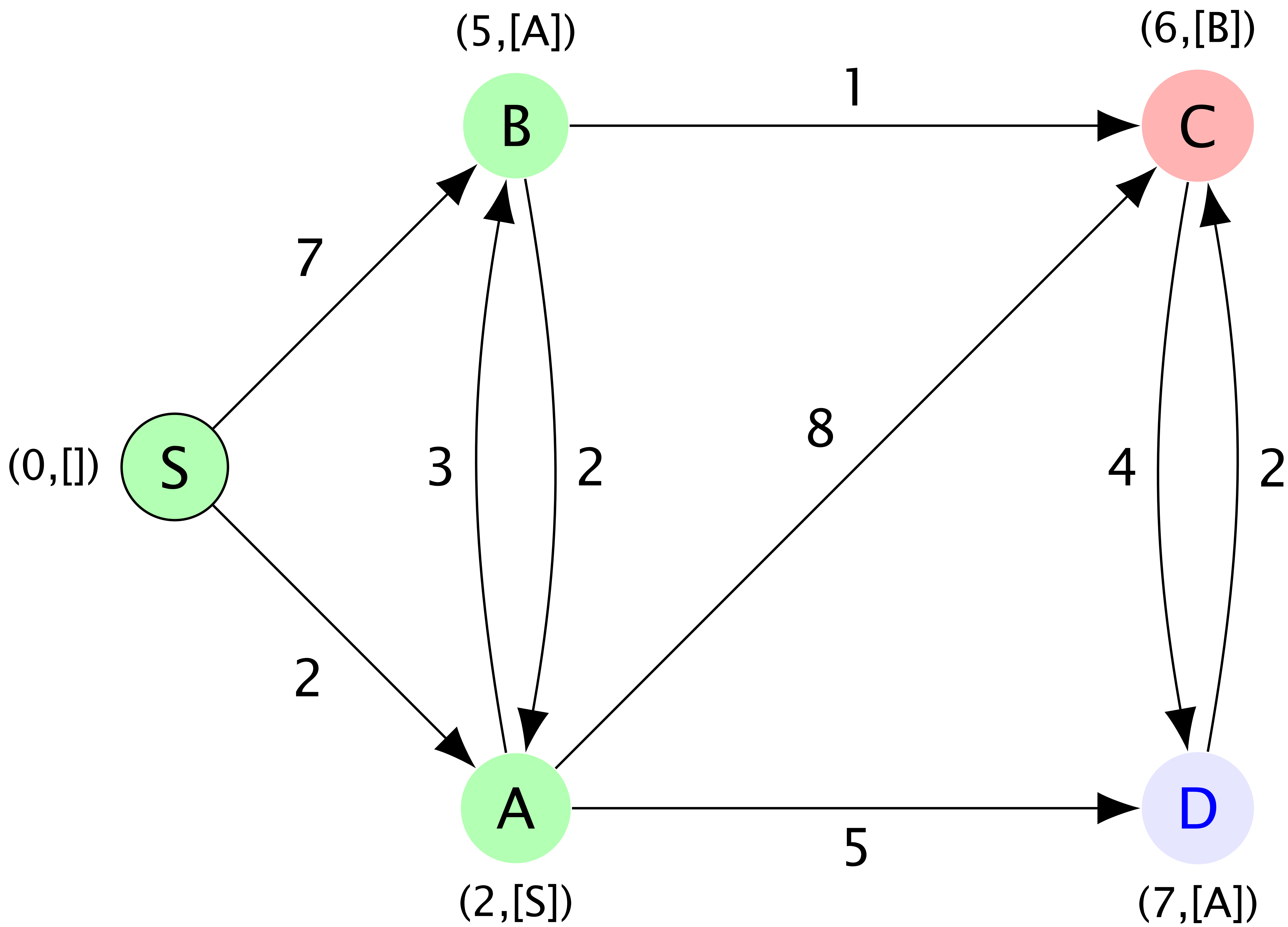
Dijkstra’s Algorithm Example

Step 3 Process B



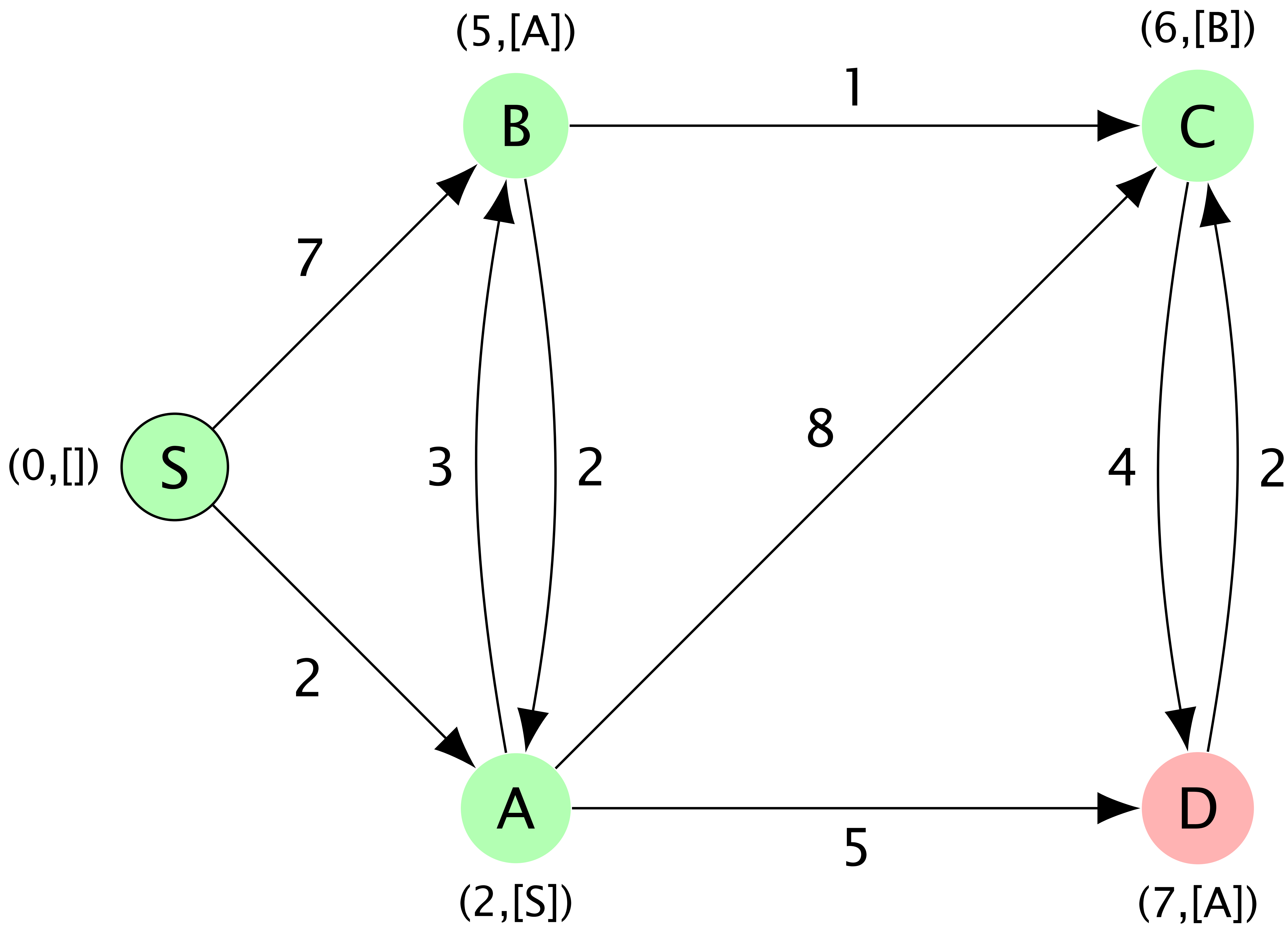
Dijkstra's Algorithm Example

Step 4 Process C



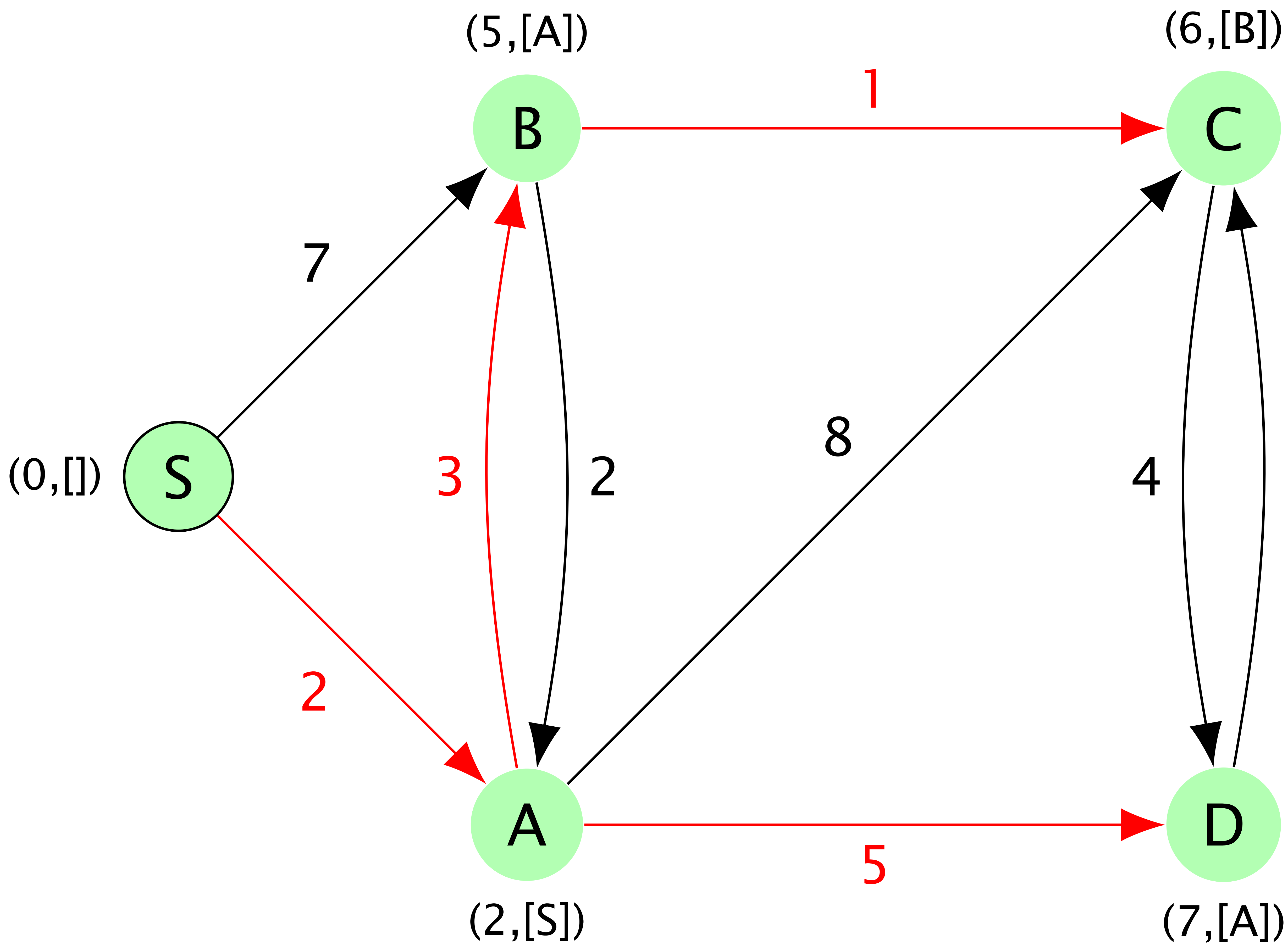
Dijkstra's Algorithm Example

Step 5 Process D



Dijkstra's Algorithm Example

Shortest Path Tree Edges

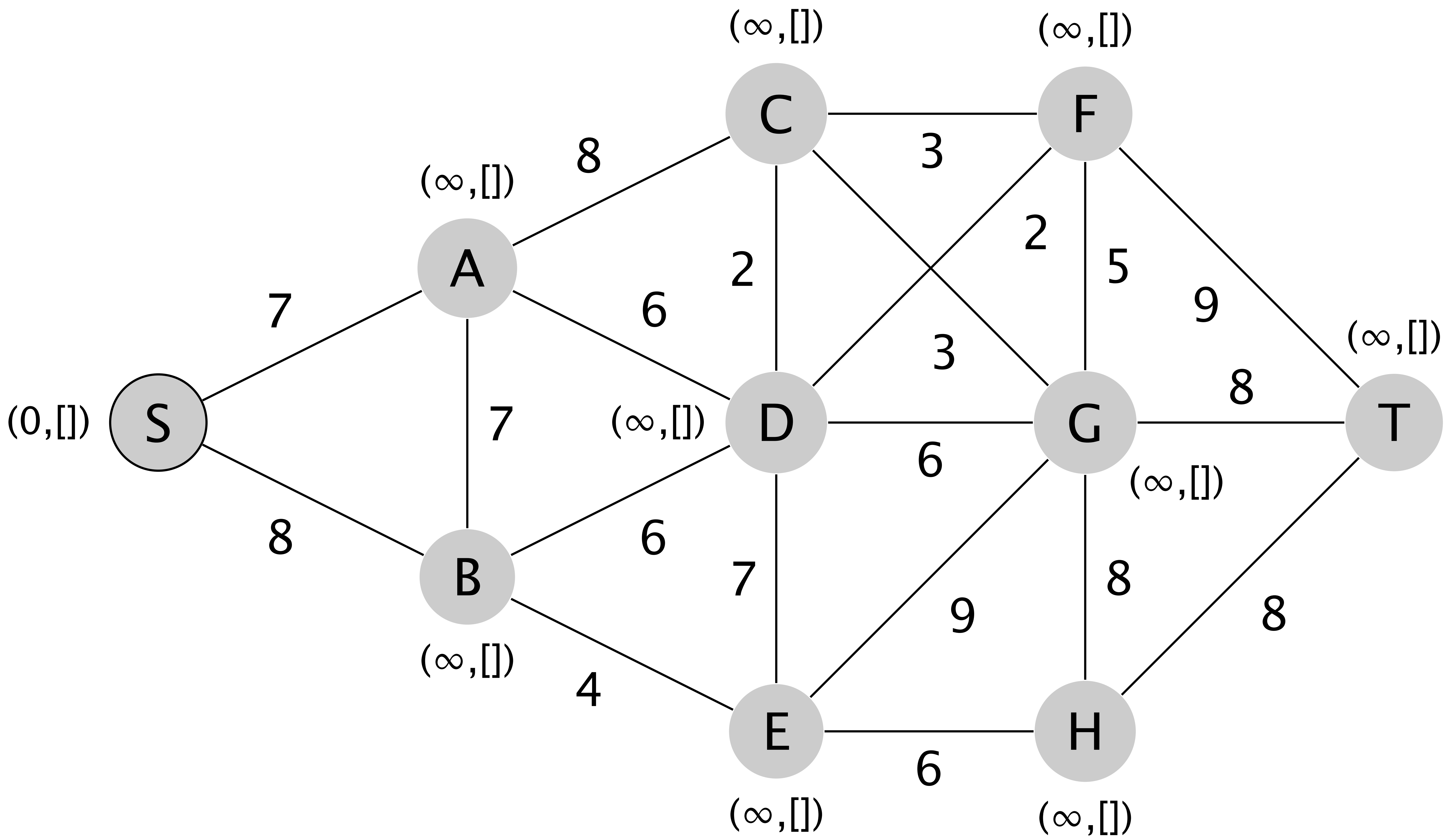


Dijkstra’s Algorithm
Further points

- See presentation at http://www.ukuug.org/events/agm2010/ShortestPath.pdf
- The algorithm as given assumes unique shortest paths — what if there are multiple shortest paths ? Modify the algorithm to accommodate this — change the weight on some edge to test this in the above example (change the weight of edge (A,C) to 4, for example)
- Implement a priority queue for Dijkstra’s algorithm
- Material essentially comes from Cormen, chp 24

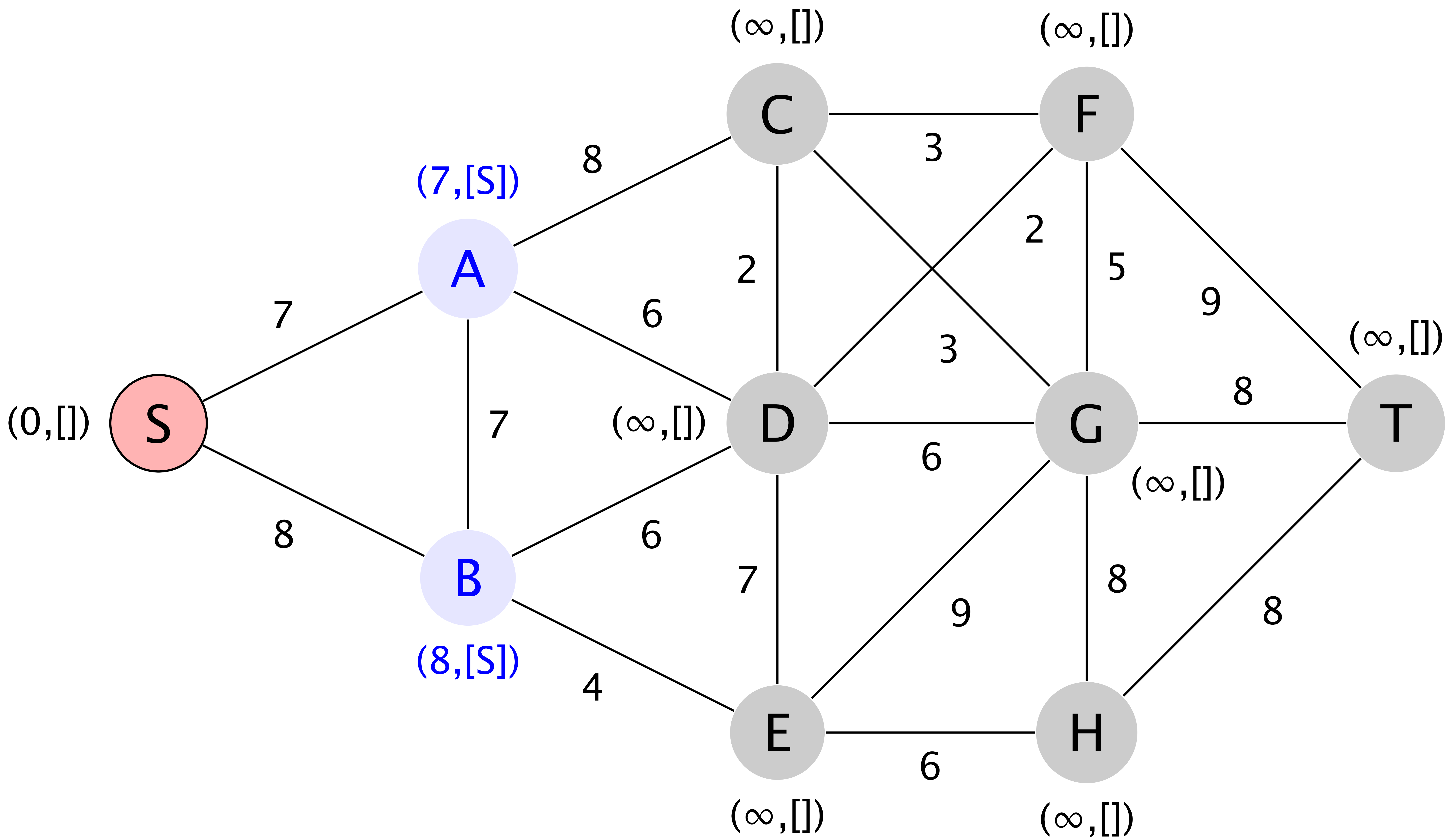
Dijkstra's Algorithm Example 02

Step 0 Initialisation



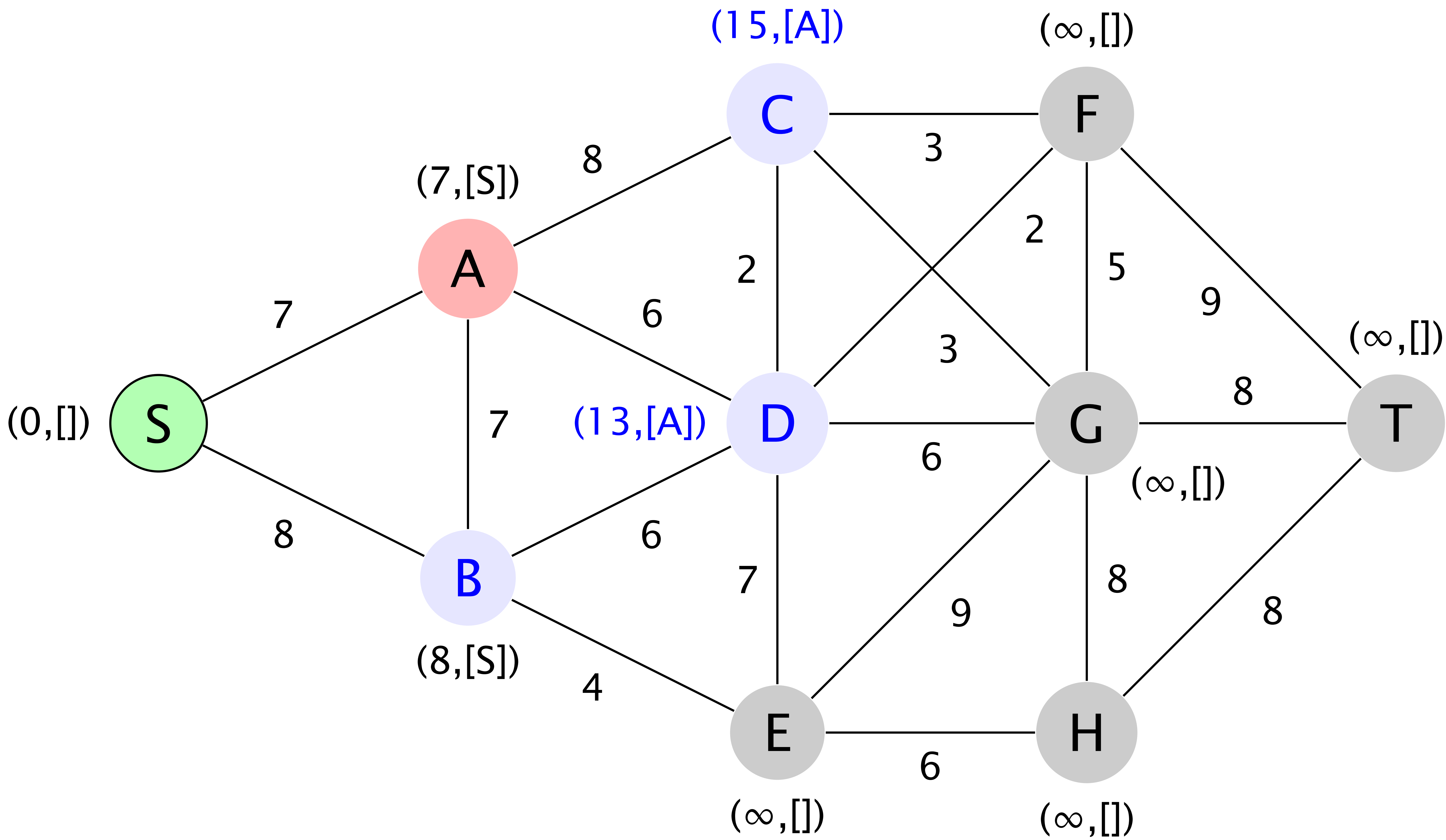
Dijkstra's Algorithm Example 02

Step 1 Process S



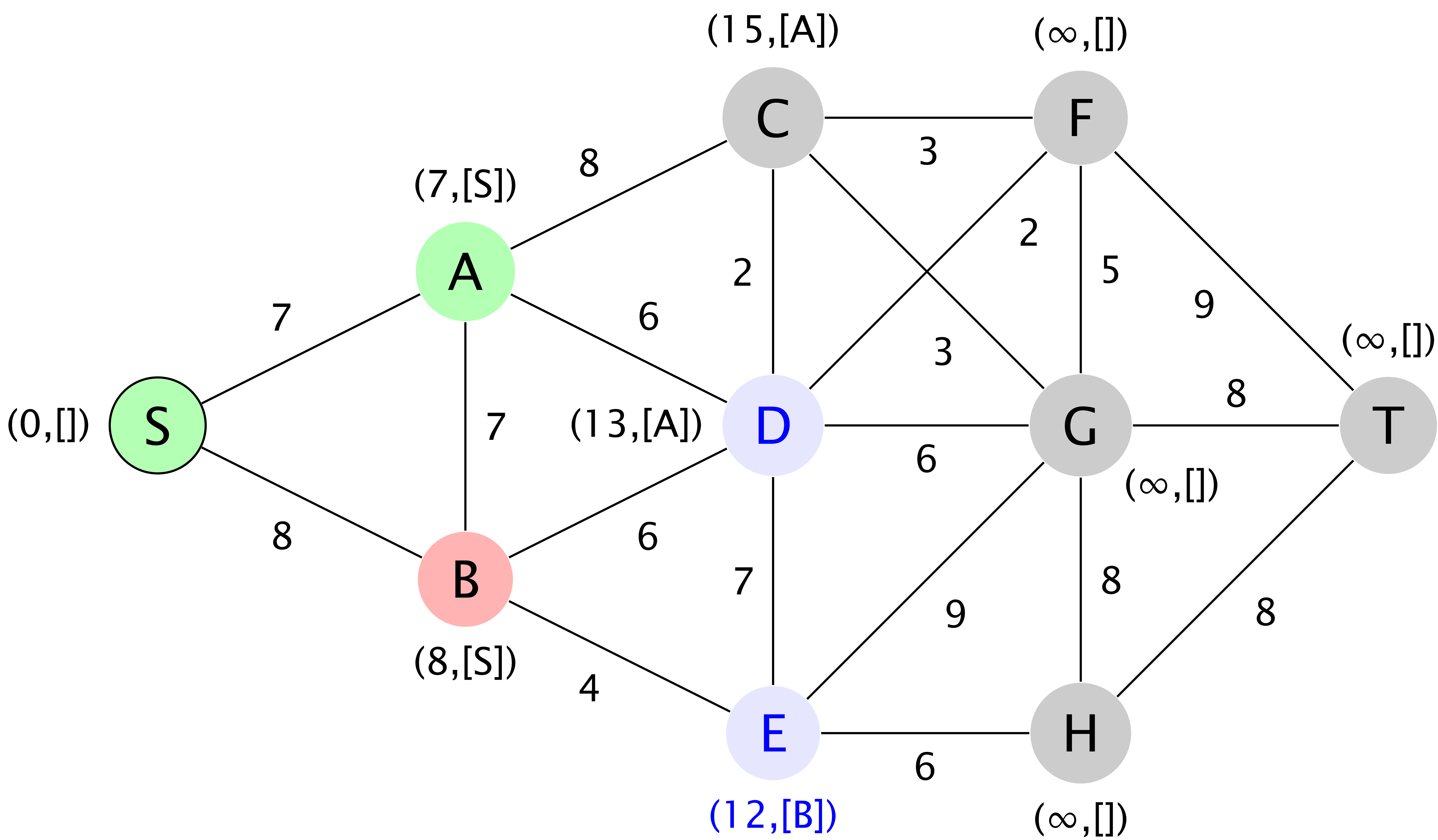
Dijkstra's Algorithm Example 02

Step 2 Process A



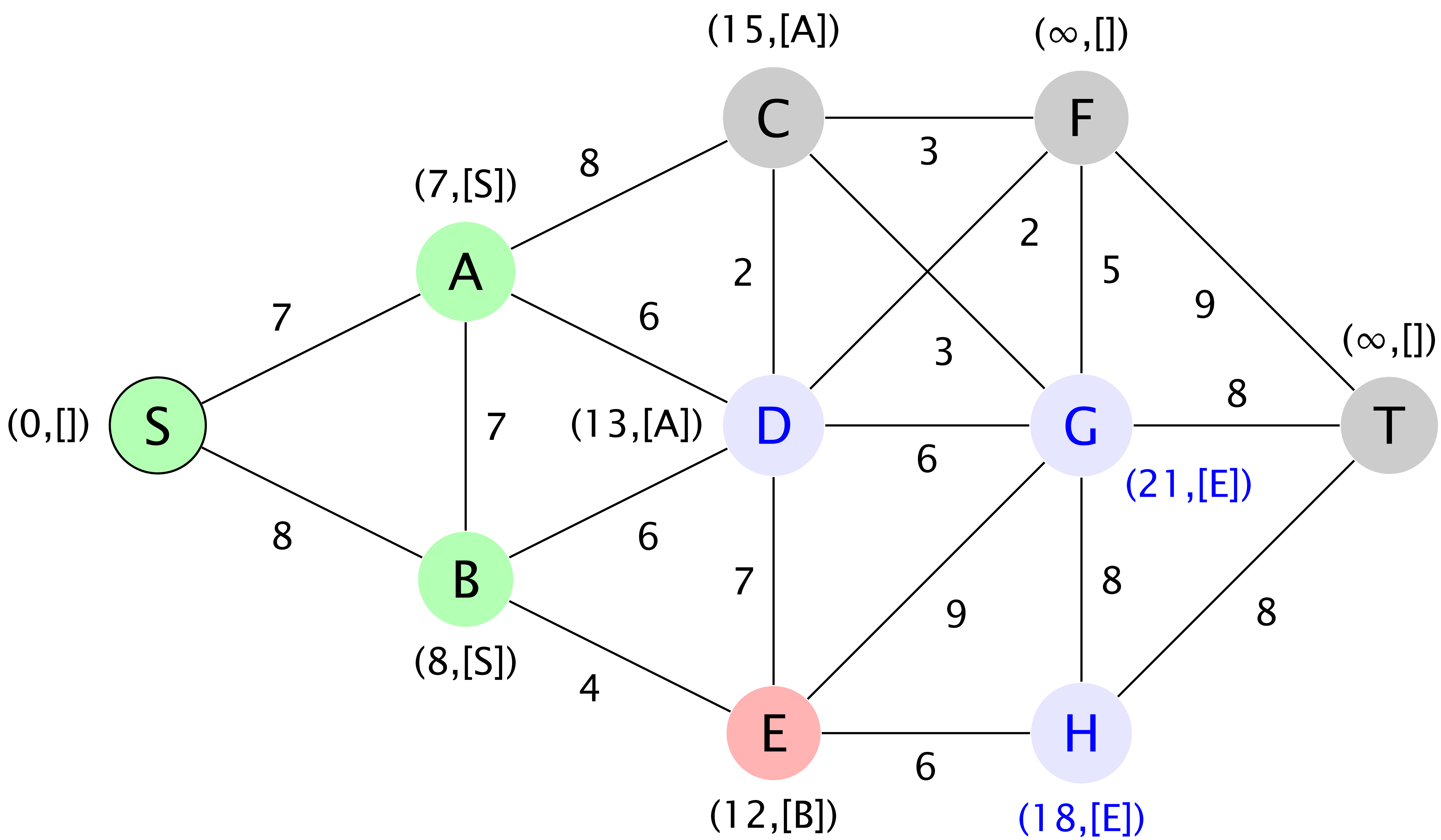
Dijkstra’s Algorithm Example 02

Step 3 Process B



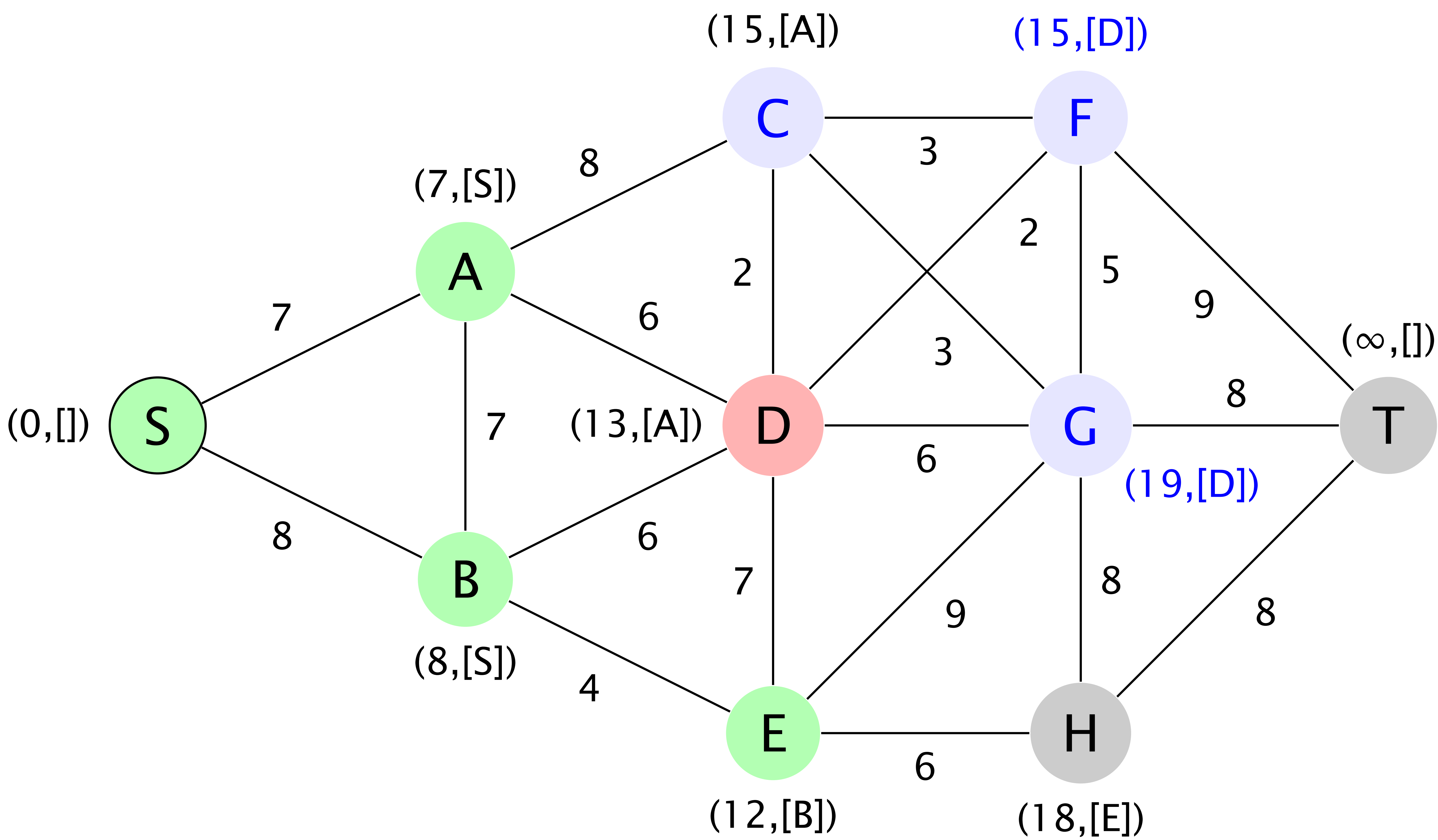
Dijkstra's Algorithm Example 02

Step 4 Process E



Dijkstra's Algorithm Example 02

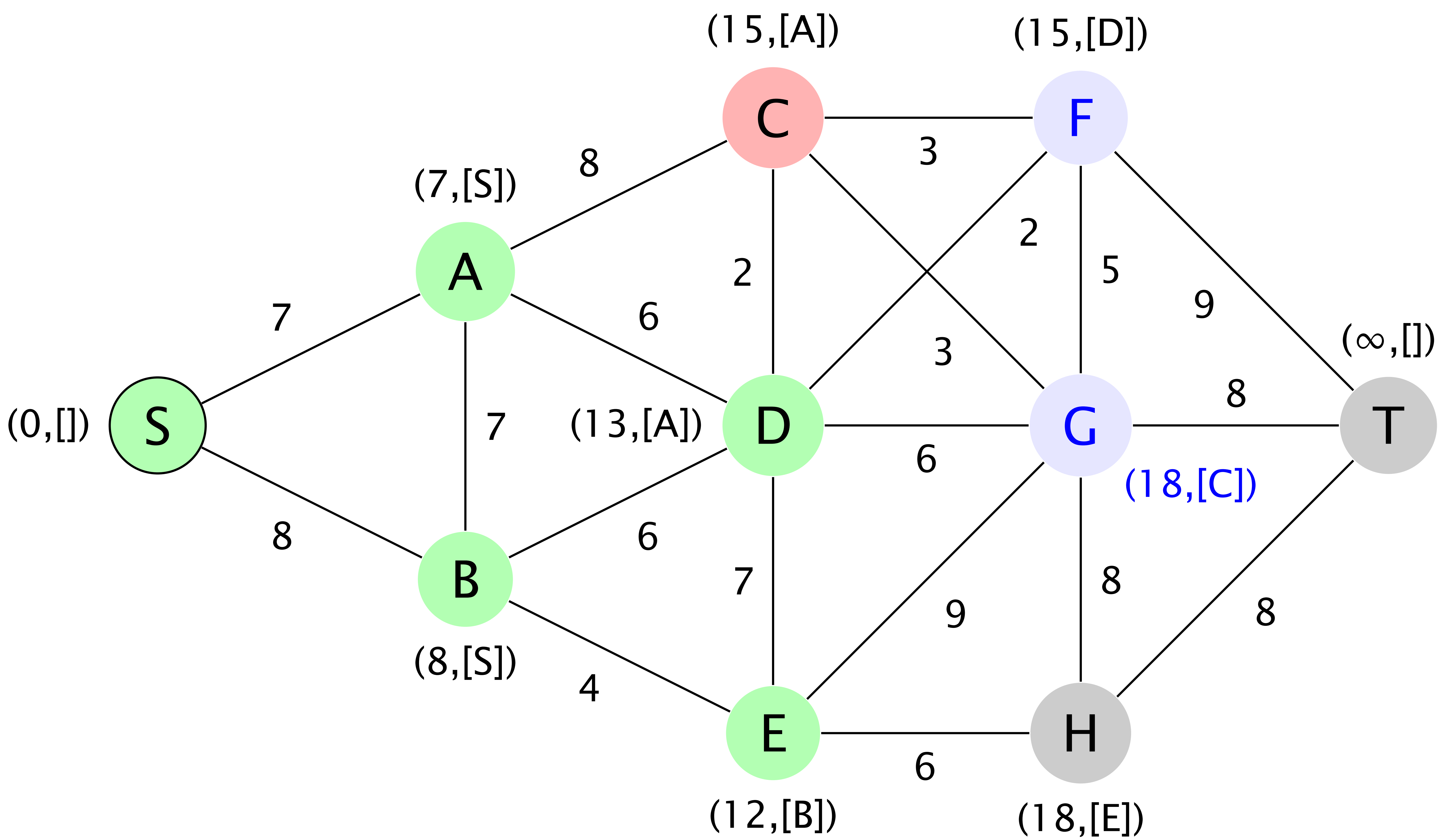
Step 5 Process D



- Vertex C should have label (15,[A,D]) if we record multiple shortest routes
- How do we change the algorithm ?

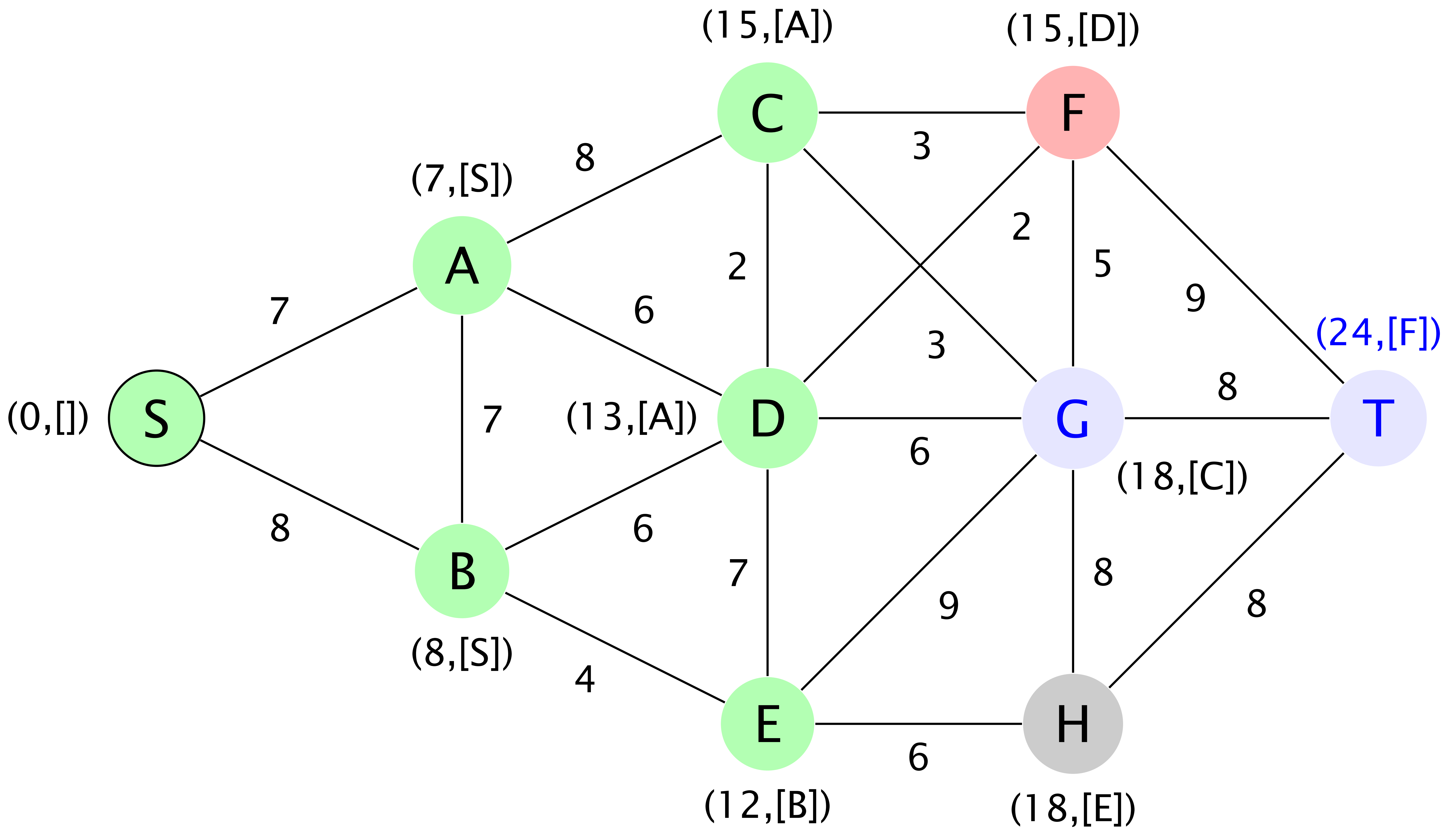
Dijkstra's Algorithm Example 02

Step 6 Process C (or F)



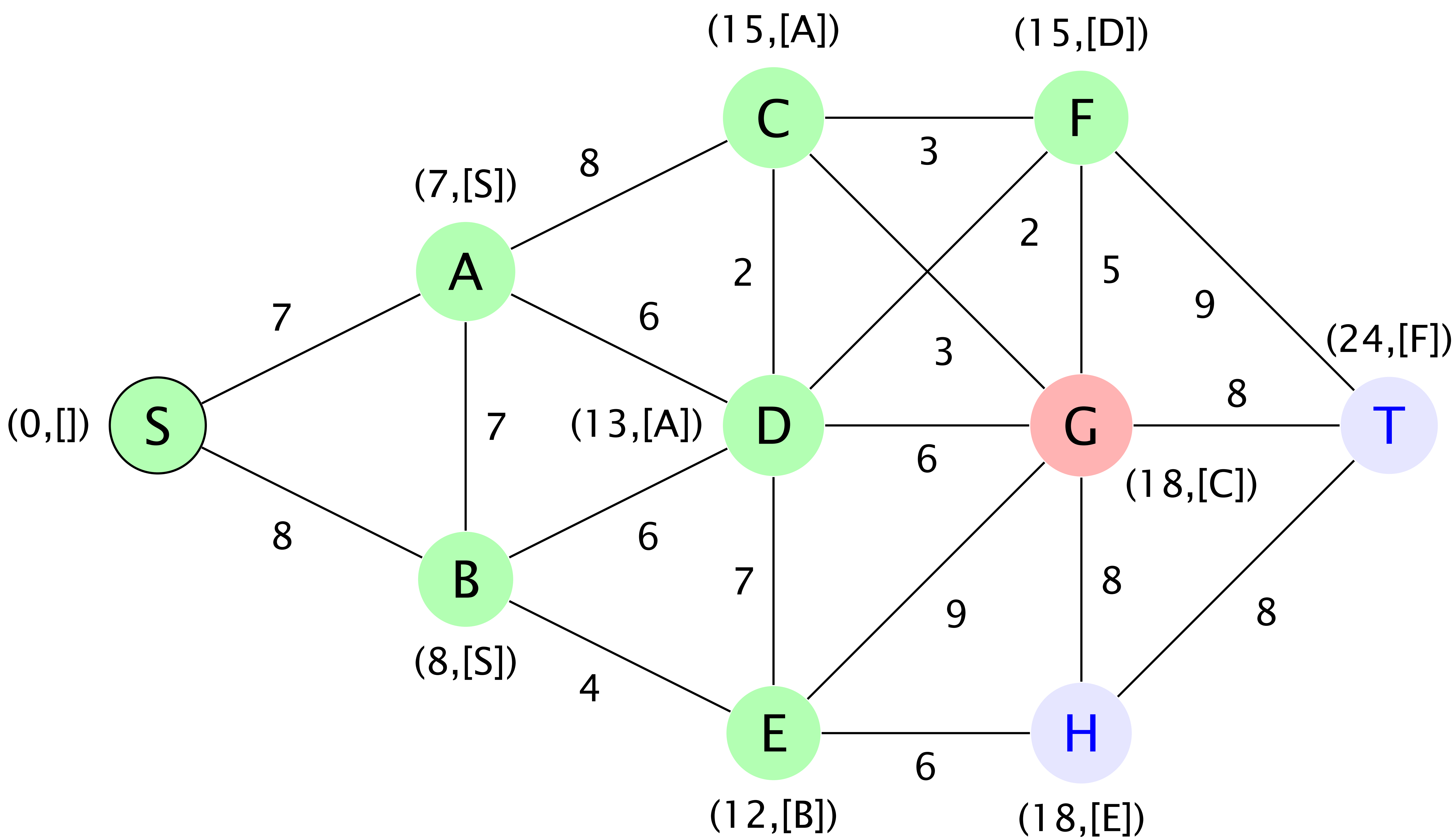
Dijkstra's Algorithm Example 02

Step 7 Process F



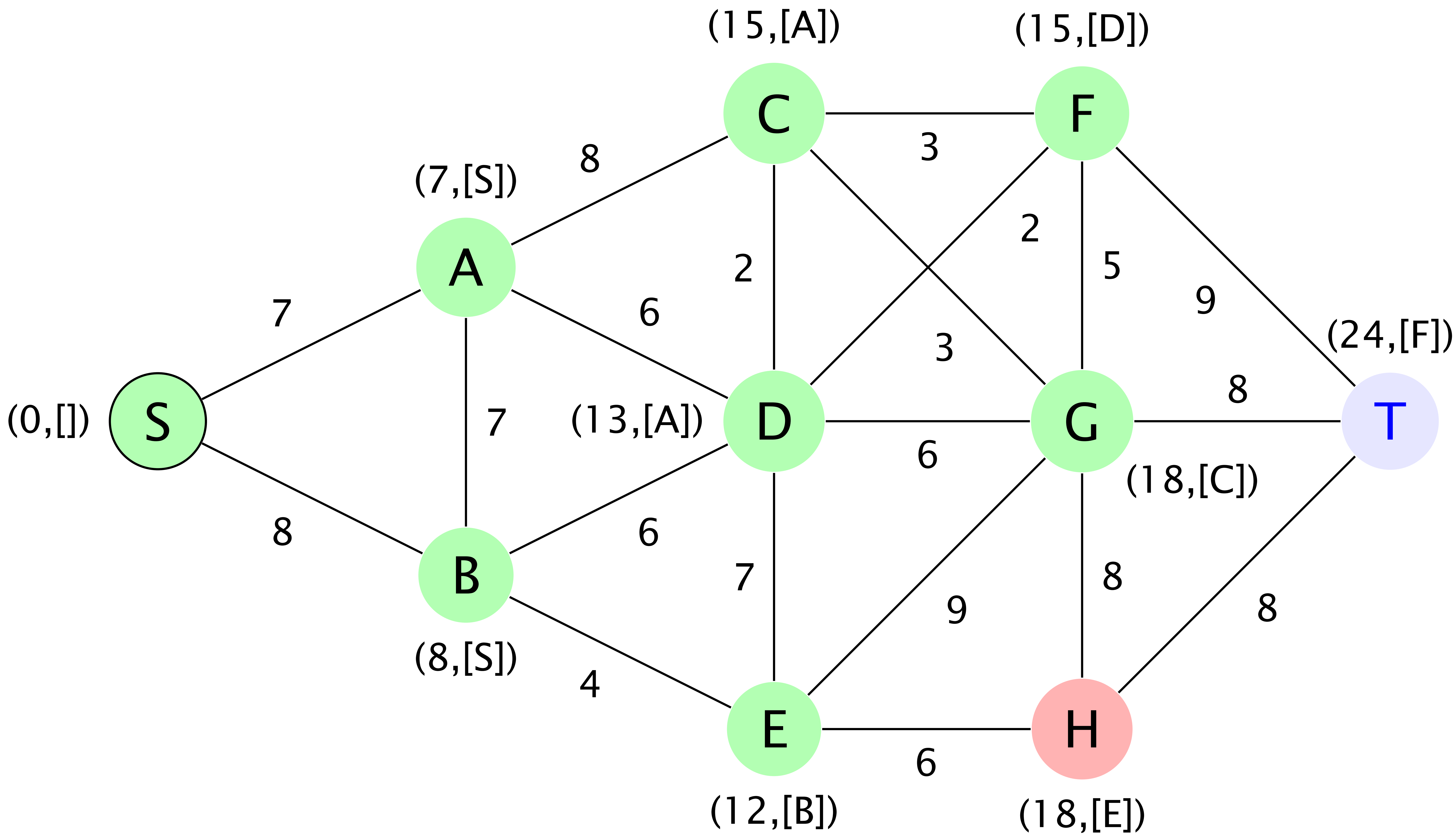
Dijkstra's Algorithm Example 02

Step 8 Process G (or H)



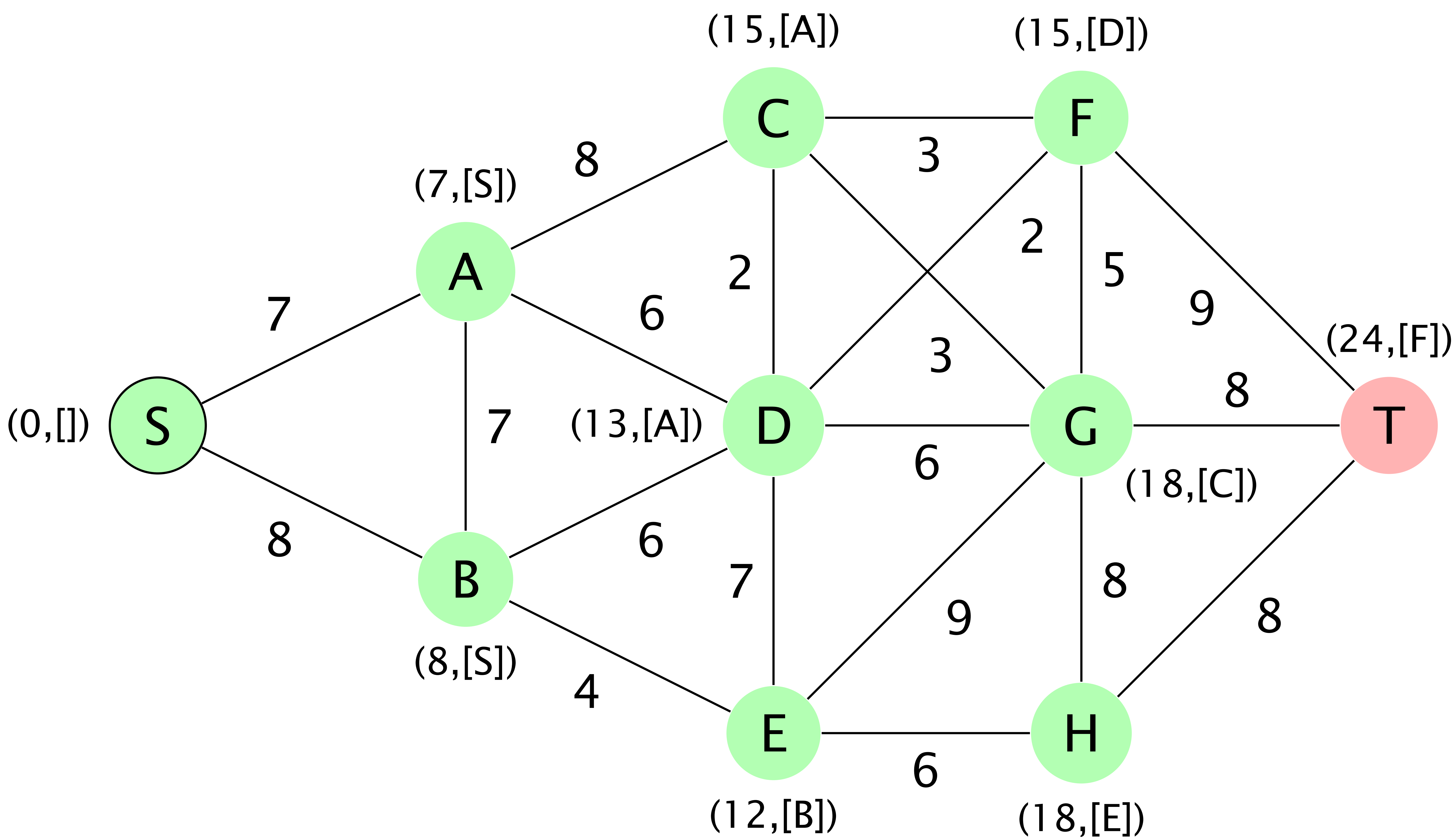
Dijkstra's Algorithm Example 02

Step 9 Process H



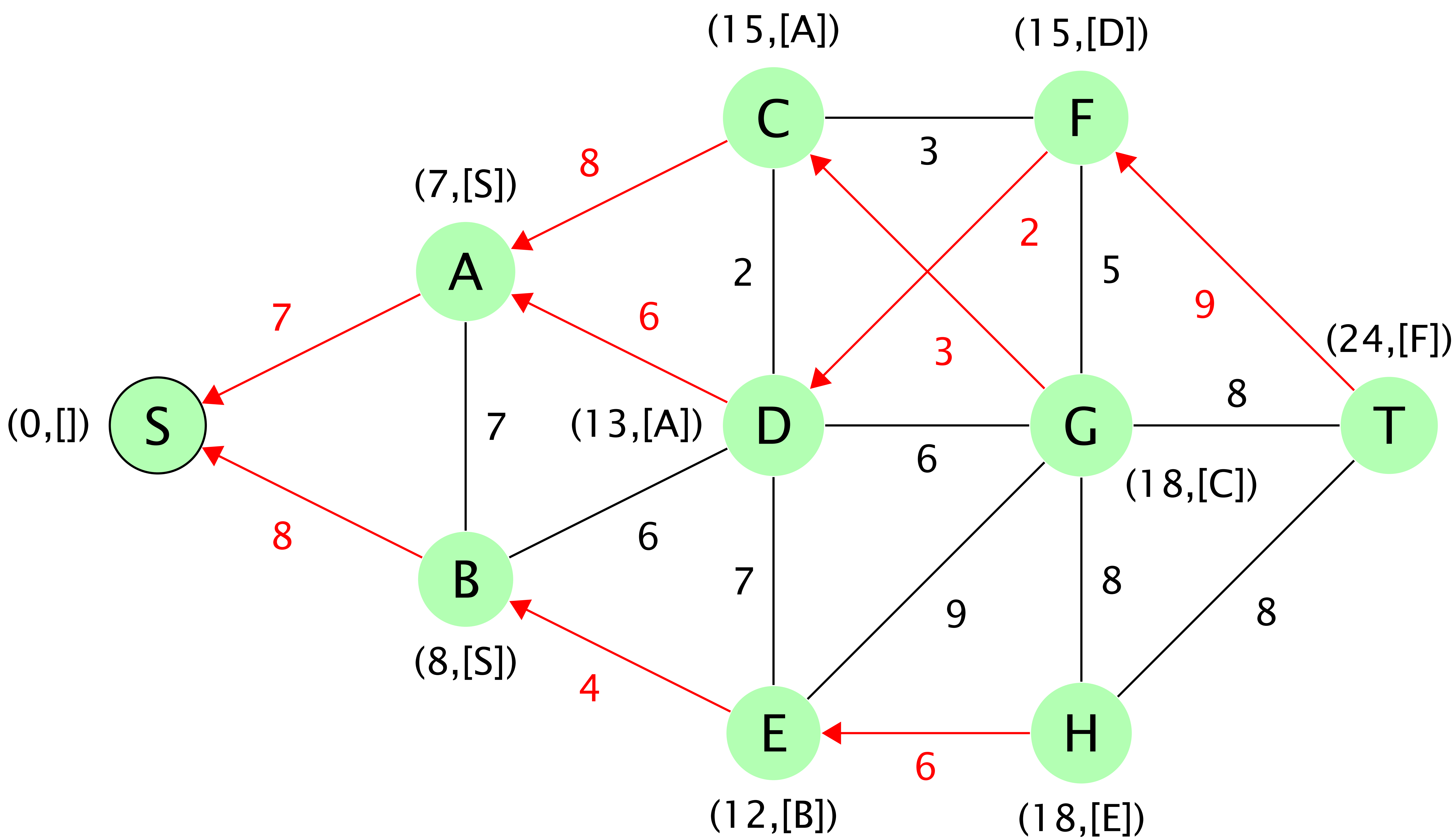
Dijkstra's Algorithm Example 02

Step 10 Process T



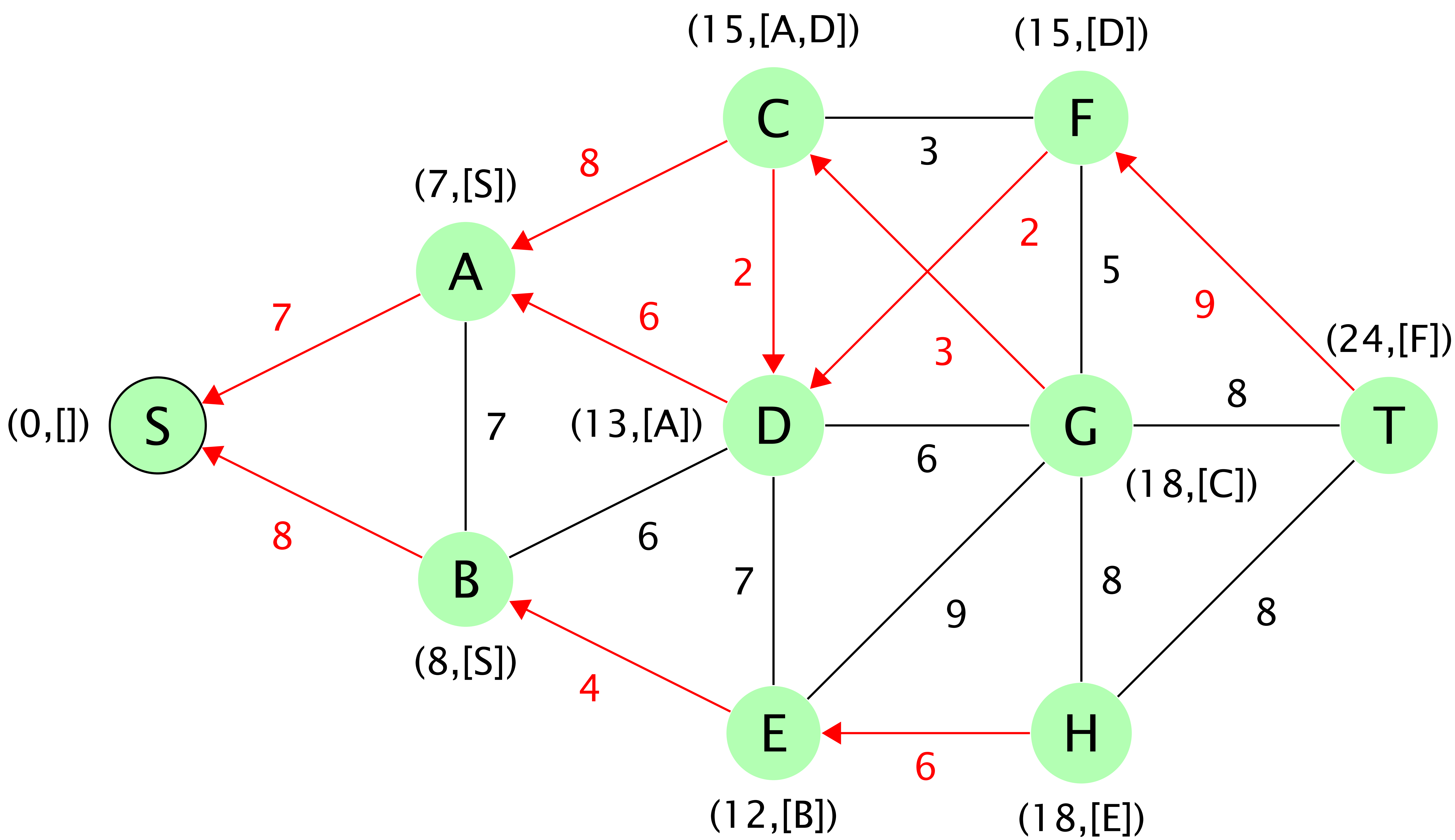
Dijkstra's Algorithm Example 02

Shortest Path Tree



Dijkstra’s Algorithm Example 02

Shortest Path Graph



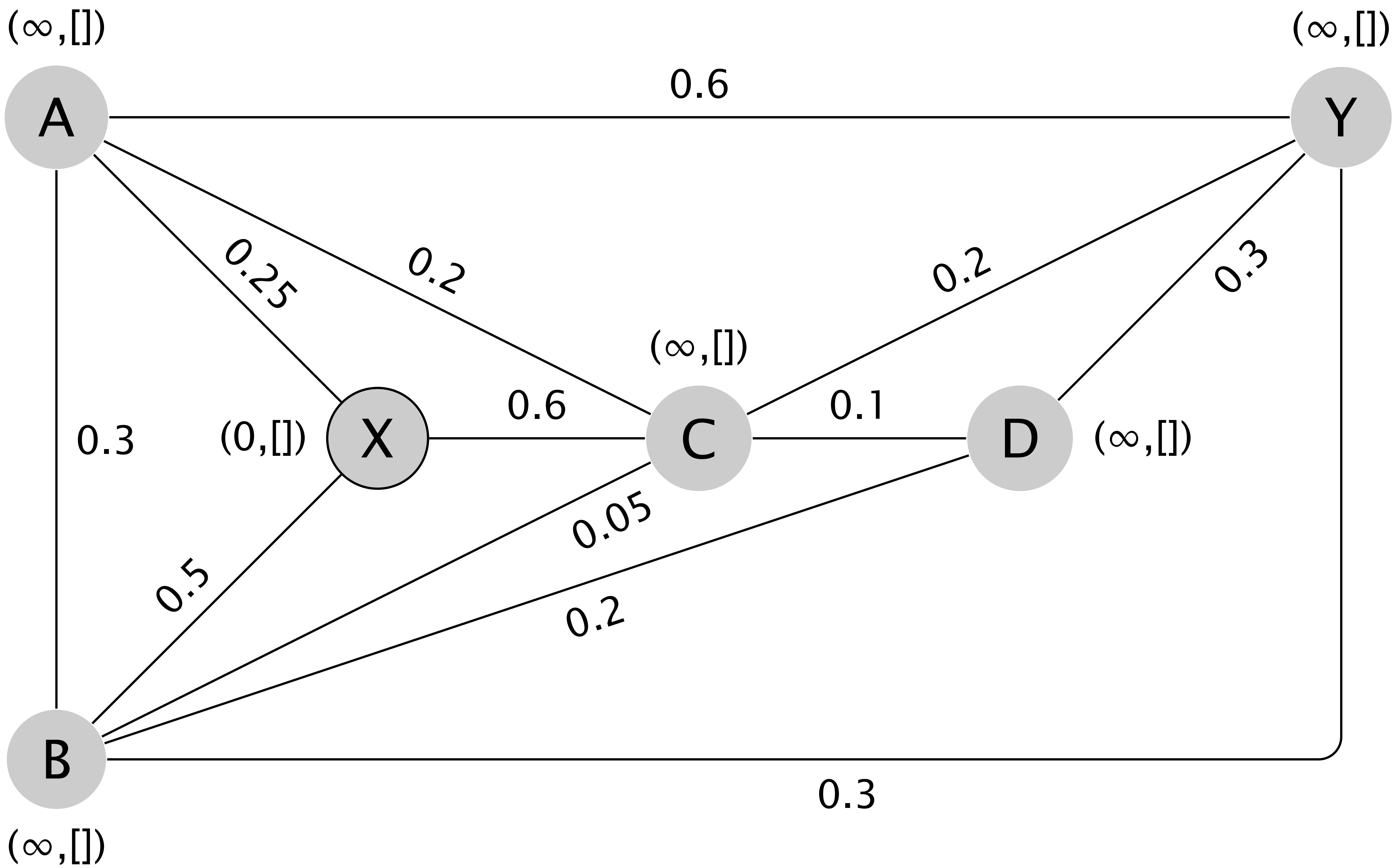
Dijkstra's Algorithm Example 03

Problem Description

- In the following graph, the weight on each edge represents the probability of failing while traversing the edge
- *Problem:* find the path that maximises the chance of traversing from X to Y

Dijkstra's Algorithm Example 03

Step 0 Initialisation



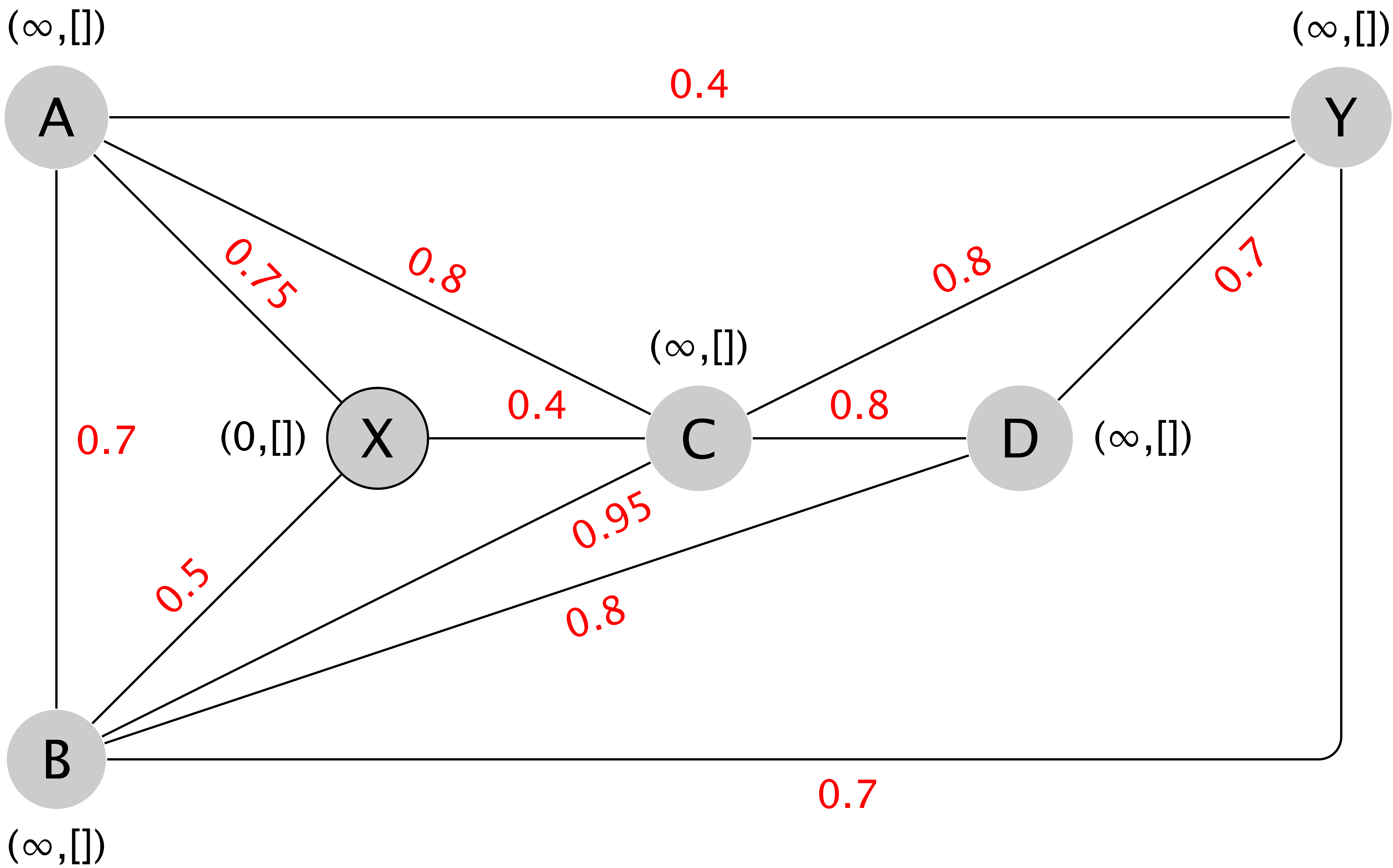
Dijkstra's Algorithm Example 03

Formulation as Shortest Path

- Let $p_{(i,j)}$ be probability of failing on edge (i, j)
- The probability of not failing is $x_{(i,j)} = 1 - p_{(i,j)}$
- Over any path $x_{(i,j)}$ are independent so problem is to maximise probability of not failing $\prod_{(i,j) \in \text{path}} x_{(i,j)}$
- Equivalently, if $y_{(i,j)} = \log x_{(i,j)}$ then problem is to maximise $\sum_{(i,j) \in \text{path}} y_{(i,j)}$
- Alternatively, since $y_{(i,j)} \in (-\infty, 0]$ as $x_{(i,j)} \in [0, 1]$ then let $z_{(i,j)} = -100 y_{(i,j)}$ and minimise $\sum_{(i,j) \in \text{path}} z_{(i,j)}$

Dijkstra's Algorithm Example 03

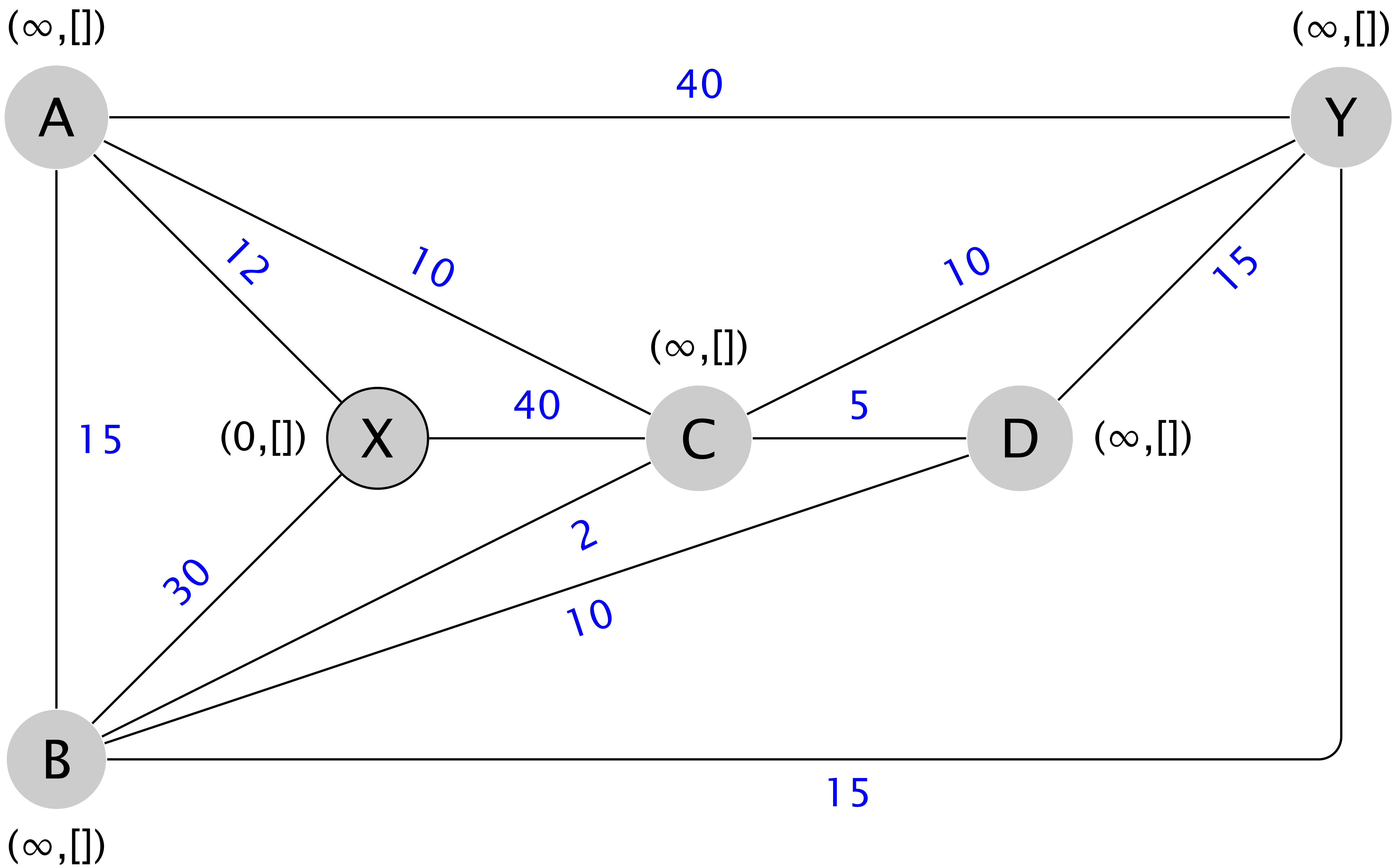
Step 0 Reformulation (a)



- The numbers in red are the probabilities of not failing
- $x_{(i,j)} = 1 - p_{(i,j)}$

Dijkstra's Algorithm Example 03

Step 0 Reformulation (b)



- The numbers in blue are negated scaled logs of $x_{(i,j)}$
- $z_{(i,j)} = -100 \log_{10} x_{(i,j)}$

Prim's Algorithm

Structured English

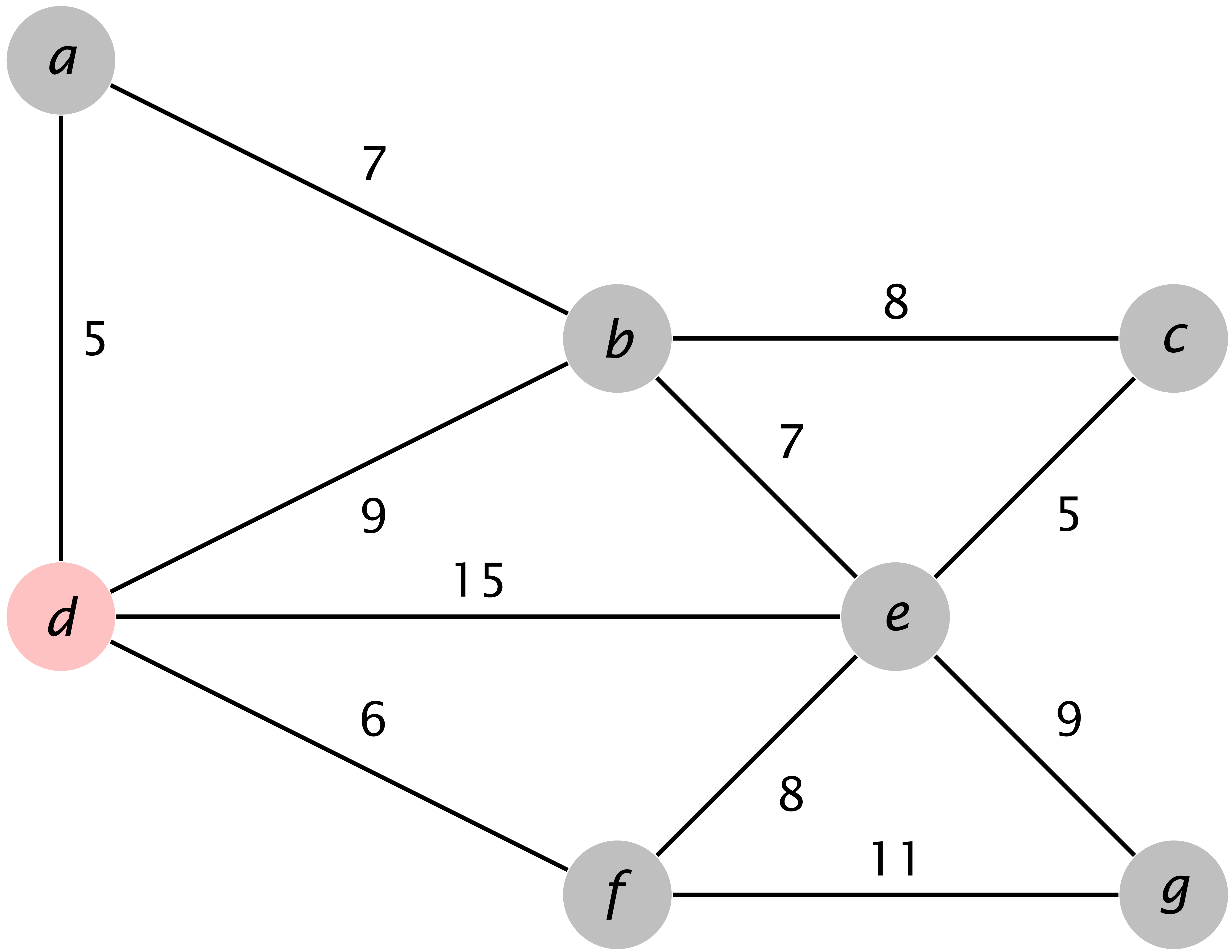
```
prim(gr,weight,r)
  for u in vertices(gr)
    key(u) = Infinity
    label(u) = Temp
  key(r) = 0
  pred(r) = None
  q = makePriorityQ(vertices(gr))

  while not isEmptyPQ(q)
    u = extractMinPQ(q)
    for v in adj(gr,u)
      if (label(v) == Temp
          and weight(edge(u,v)) < key(v))
        key(v) = weight(edge(u,v))
        q = decreaseKeyPQ(q,v,key(v))
        pred(v) = u
    label(u) = Permanent
```

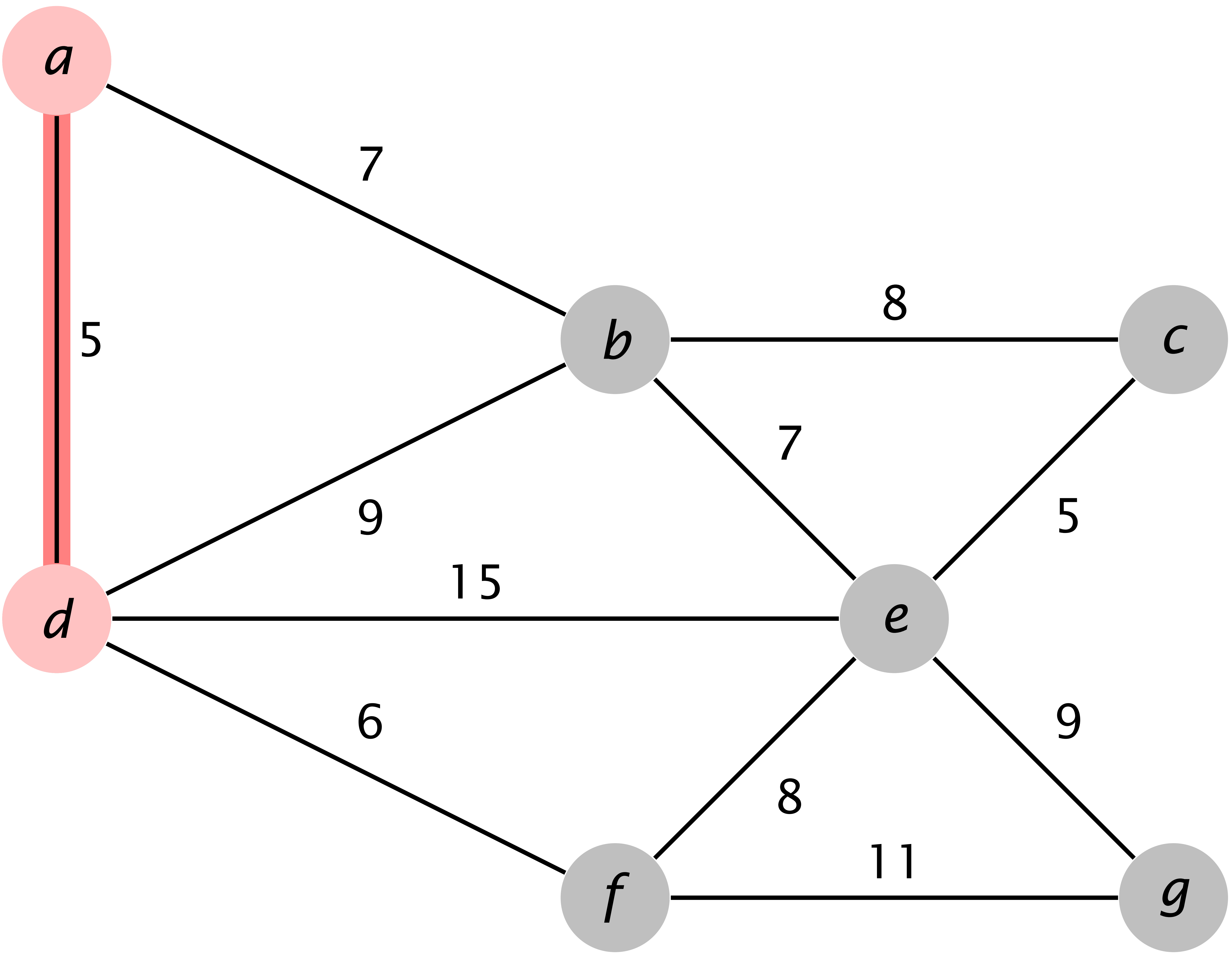
Dijkstra's and Prim's Algorithms
Comparison

- Both are examples of *greedy* algorithms
- They choose the next best edge to add to the permanently labelled set
- The algorithms are very similar
- Process each vertex, v, in turn from a priority queue
- Examine all vertices adjacent to v and perform *relaxation*
- *relaxation* means updating the distances or keys
- For the term *relaxation* see Cormen (2009, page 648) has a footnote explaining the origin of the term *relaxation*

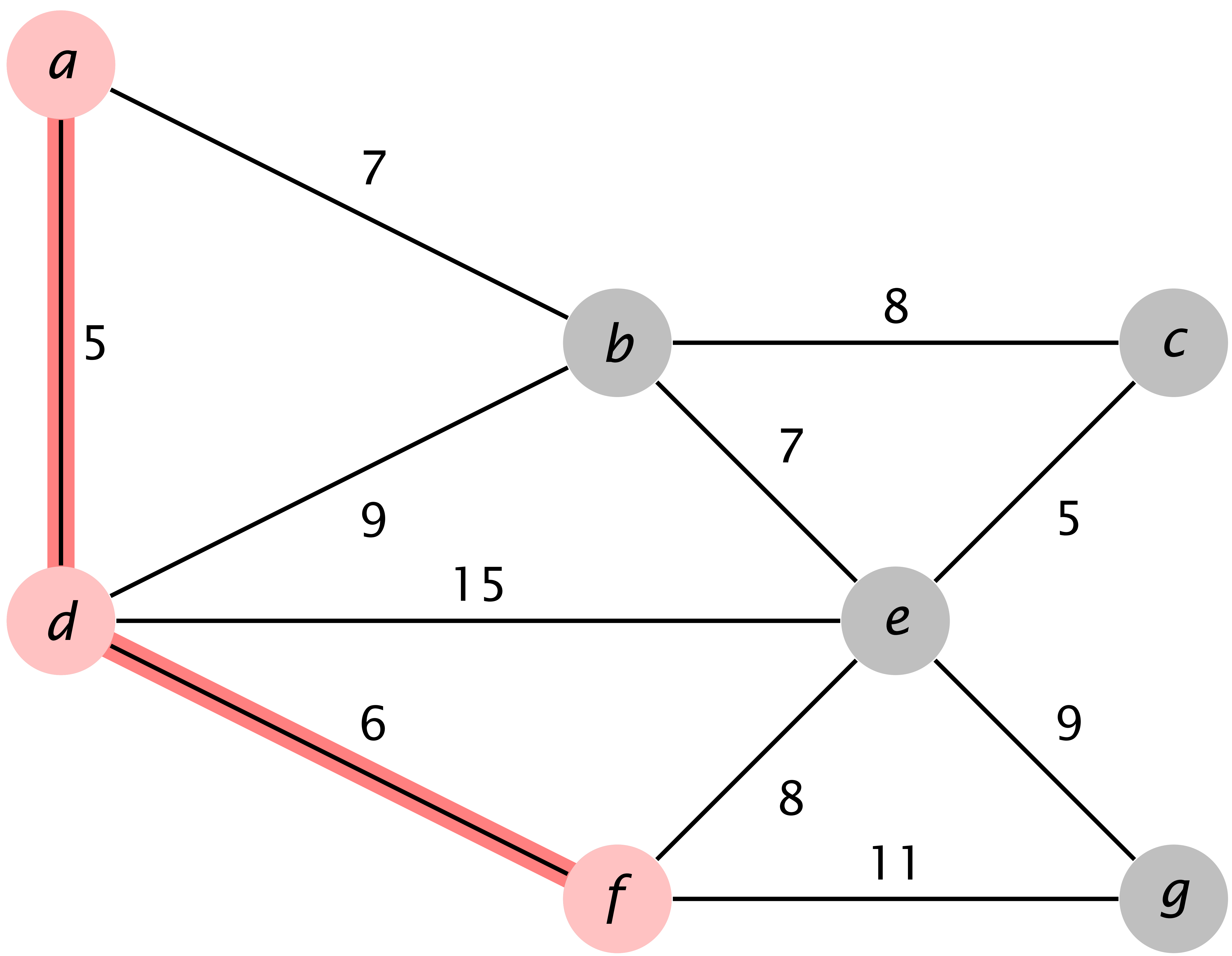
Tutorial Material — Prim's algorithm



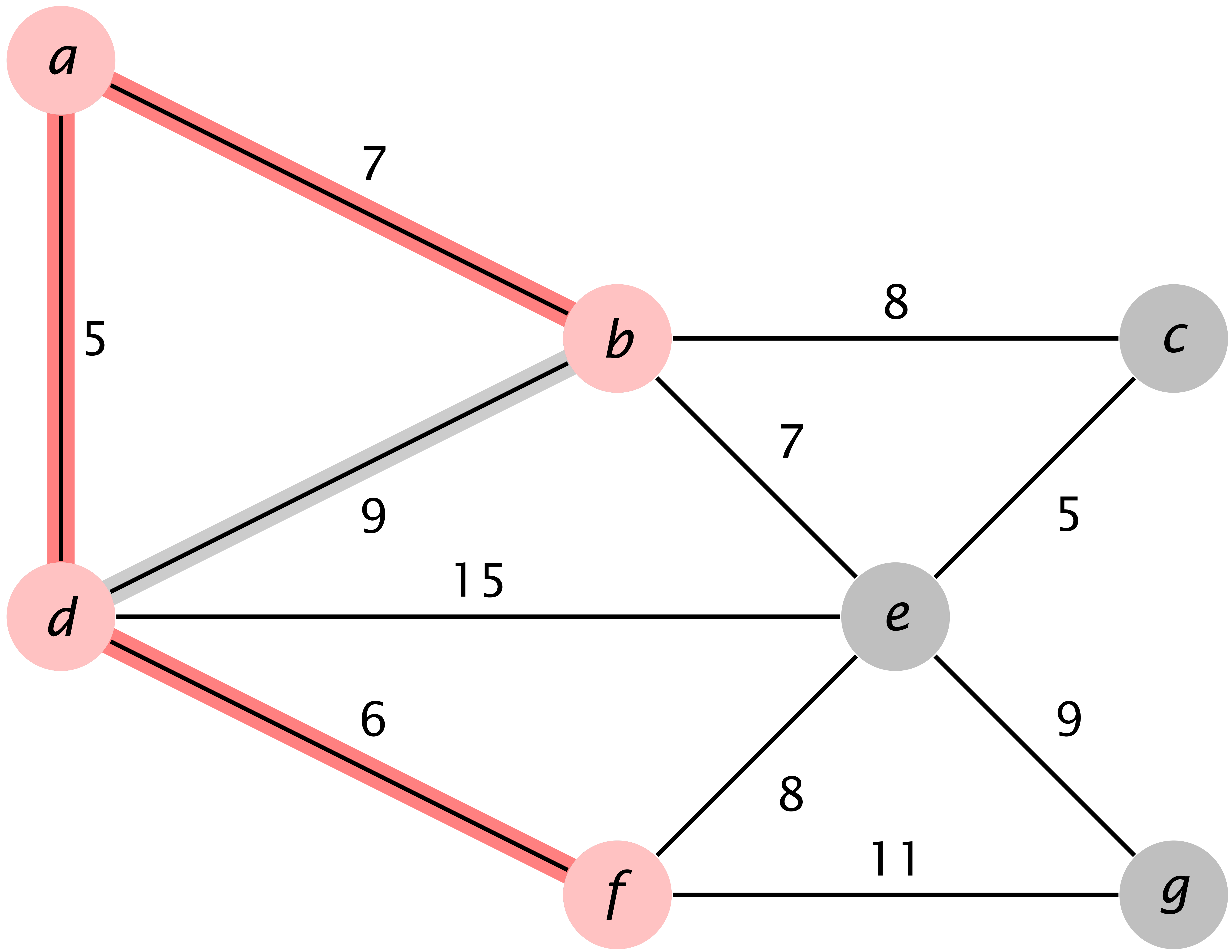
Tutorial Material — Prim's algorithm



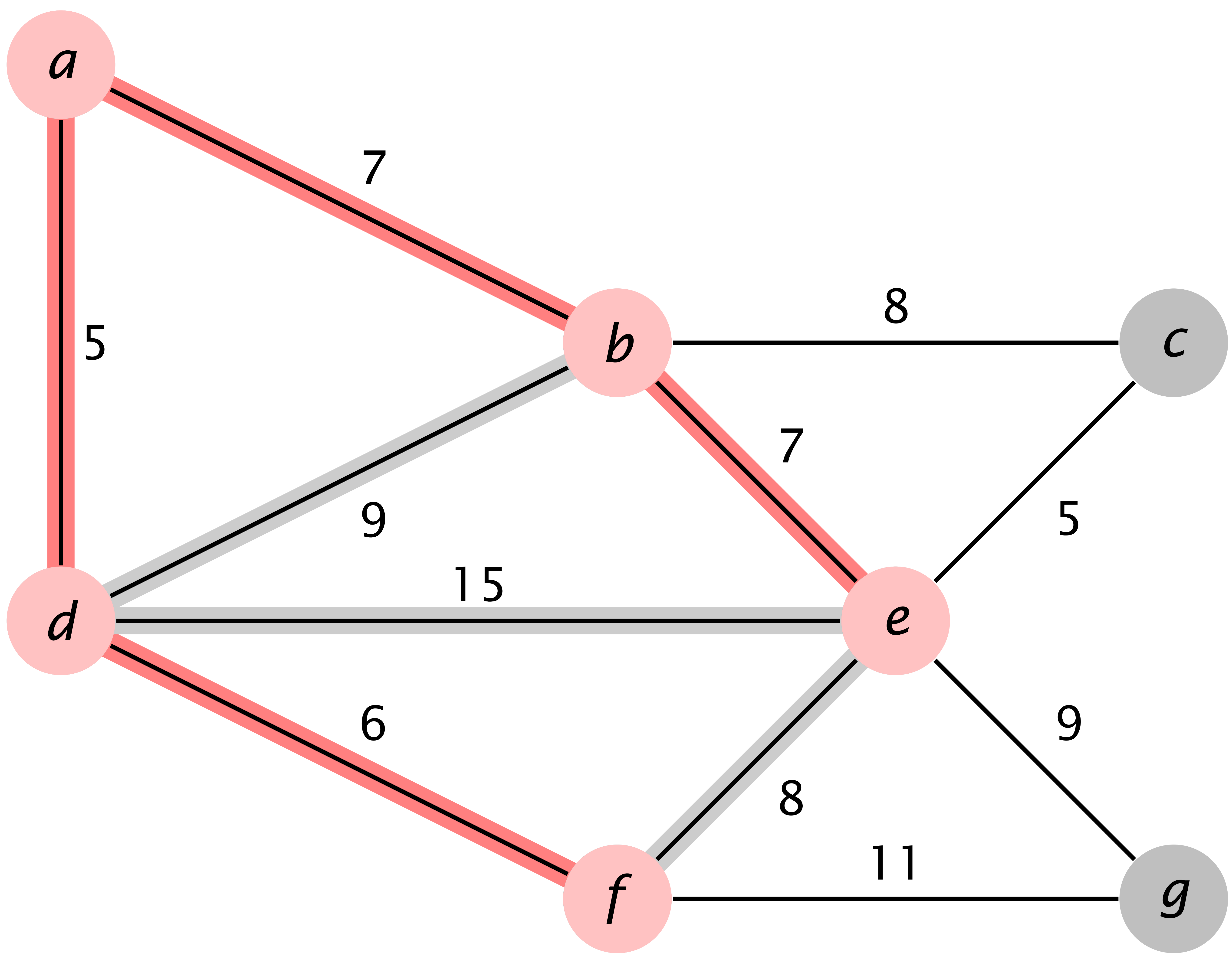
Tutorial Material — Prim's algorithm



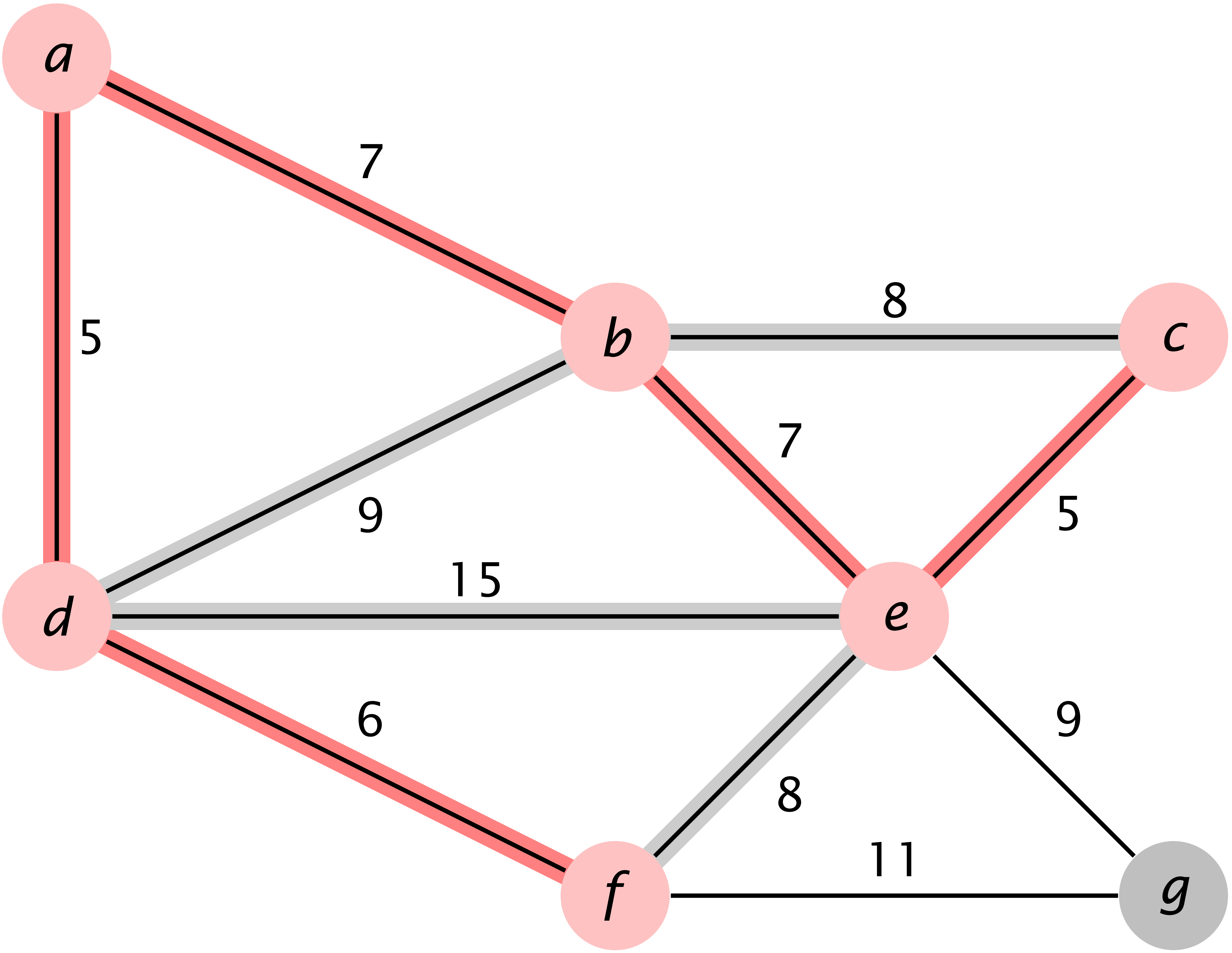
Tutorial Material — Prim's algorithm



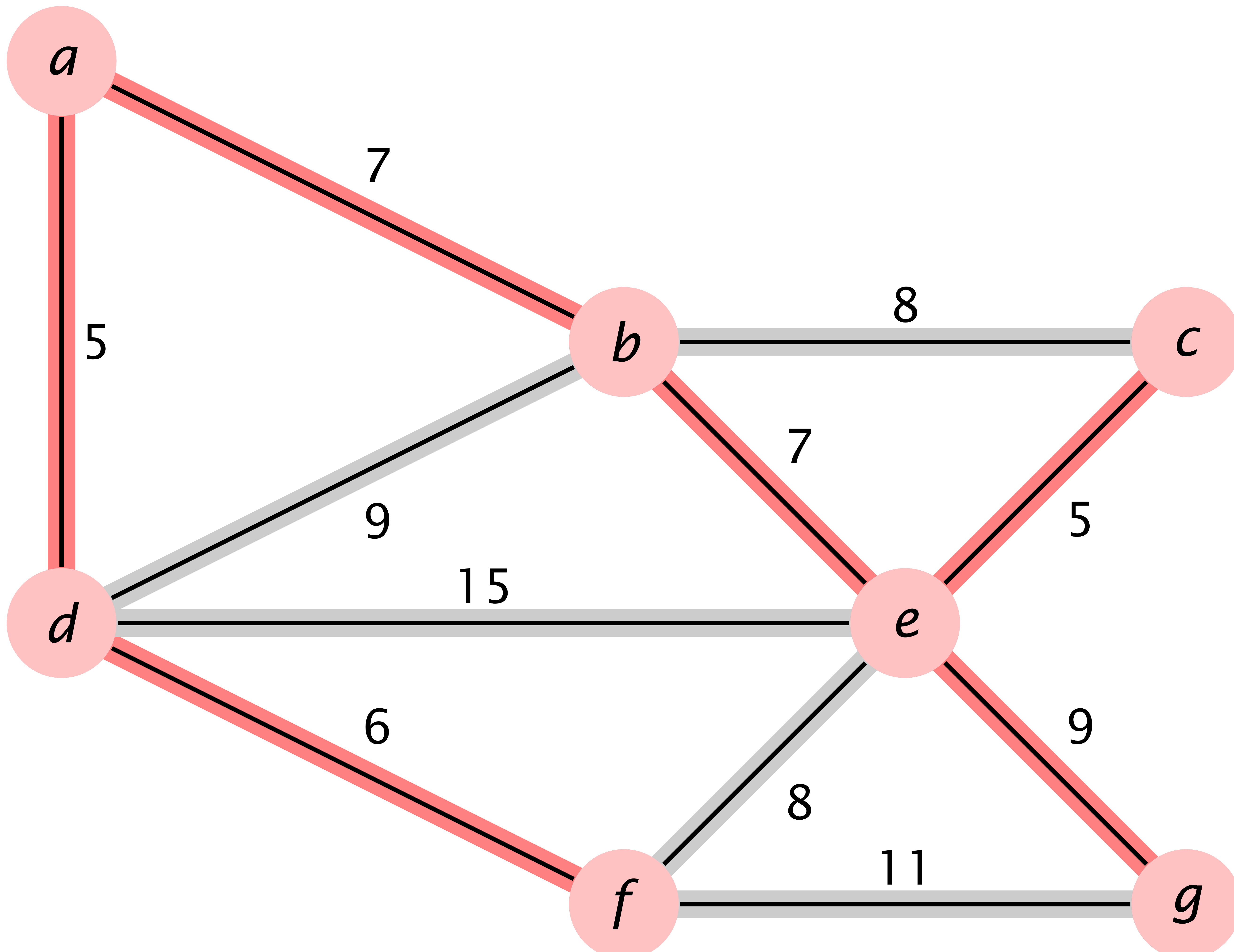
Tutorial Material — Prim's algorithm



Tutorial Material — Prim's algorithm



Tutorial Material — Prim's algorithm



Dynamic Programming

Fibonacci Sequence

- The Fibonacci Numbers or Sequence invented by Leonardo Fibonacci Pisano, who also popularized the Hindu-Arabic numeral system via his 1202 book *Liber Abaci (Book of Calculations)*
- Defined by the recurrence relation
- $F_n = F_{n-1} + F_{n-2}$
- $F_0 = 0, \quad F_1 = 1$ (or originally, $F_1 = 1, \quad F_2 = 1$)
- The Fibonacci numbers have lots of interesting properties
- In programming, often used to illustrate ideas about recurrence relations and recursion

Fibonacci Sequence

Python Recursive (1)

- Recursive definitions are recursive algorithms

```
def fibs1(n):
  return fibs1indent(n,0)

def fibs1indent(n,indent):
  indentStr = ' '*indent
  print(indentStr + 'fibs(' + str(n) + ')')

  if n == 0 :
    return 0
  elif n == 1 :
    return 1
  else:
    return (fibs1indent(n - 2, indent + 2)
            + fibs1indent(n - 1, indent + 2))
```

Fibonacci Sequence

Python Recursive (2)

```
Python3>>> fibs1(5)
fibs(5)
  fibs(3)
    fibs(1)
    fibs(2)
      fibs(0)
      fibs(1)
  fibs(4)
    fibs(2)
      fibs(0)
      fibs(1)
    fibs(3)
      fibs(1)
      fibs(2)
        fibs(0)
        fibs(1)
5
Python3>>>
```

- This take 15 calls to `fibs1()`, 5 calls to `fibs(1)`, 3 calls to `fibs(0)`

Fibonacci Sequence

Python Recursive (2)

- Recursion but remembering enough to avoid most

```python
def fibs2(n):
  return fibs2indent(n,0)

def fibs2indent(n,indent,first=0,second=1):
  indentStr = ' '*indent
  print(indentStr + 'fibs(' + str(n) + ')')

  if n == 0 :
    return [first]
  else:
    return ([first]
              + fibs2indent(n - 1, indent + 2
                             , second, first + second))
```

Fibonacci Sequence

Python Recursive (2)

```
Python3>>> fibs2(5)
fibs(5)
  fibs(4)
    fibs(3)
      fibs(2)
        fibs(1)
          fibs(0)
[0, 1, 1, 2, 3, 5]
Python3>>>
```

- This take 6 calls to `fibs2()`
- `fibs2()` $T(n) = T(n-1) + 1 = n + 1$
- `fibs1()` $T(n) = T(n-1) + T(n-2) + 1 = 2F_{n+1} - 1$
- `fibs1(1)` gets called F_n times
- `fibs1(0)` gets called F_{n-1} times
- So the recursion tree has $F_n + F_{n-1} = F_{n+1}$ leaves
- Since the tree is full it must have $2F_{n+1} - 1$ nodes
- (it is the sum of a geometric series)

Fibonacci Sequence

Closed-form Solution

- $F_n = \dfrac{\phi^n - (1 - \phi)^n}{\sqrt{5}}$
- Euler-Binet Formula
- A formula for Fib(n)
- ϕ is the Golden mean
- $\phi = \dfrac{1}{\phi - 1} = \dfrac{1 + \sqrt{5}}{2}$
- Also known as the Golden ratio
- $\phi \overset{def}{=} \dfrac{a}{b} = \dfrac{a + b}{a}$

Fibonacci Sequence

Calculation (1)

- From How to Compute Fibonacci Numbers? and Fibonacci Formulae

```
fib (n + k) = fib (n-1) * fib k + fib n * fib (k+1)
```

- Proof by induction

```
fib (n + 2 + k)
  = fib (n+k) + fib (n+k+1)                  -- by fib
  = fib (n-1) * fib k + fib n * fib (k+1)
    + fib n * fib k + fib (n+1) * fib (k+1)  -- by hyp
  = fib (n+1) * fib k + fib (n+2) * fib (k+1) -- by fib
```

Fibonacci Sequence

Calculation (2)

- We now derive a method to compute fib in $O(\log n)$

```
fib (2n+1) = (fib n)^2 + (fib (n+1))^2   -- (1)
fib (2n+2) = fib n * fib (n+1) + fib (n+1) * fib (n+2)
  = fib n * fib (n+1) + fib (n+1) * (fib n + fib (n+1))
  = 2 * fib n * fib (n+1) + (fib (n+1))^2  -- (2)
fib 2n = 2 * fib n * fib (n+1) - (fib n)^2 -- (3)
  -- (3) = (2) - (1)
```

```
fib2v :: Int -> (Int,Int)
fib2v 0 = (0,1)
fib2v n
  | n `mod` 2 == 0  = (c,d)
  | otherwise       = (d,c+d)
  where
  (a,b) = fib2v (n `div` 2)
  c     = 2 * a * b - a * a
  d     = a * a + b * b
```

Dynamic Programming

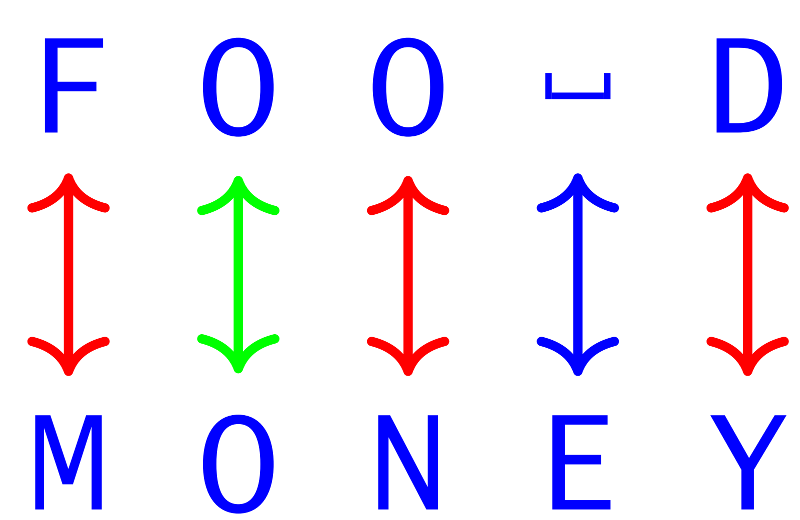
DP Formulation

- Write a recursive algorithm for the problem
- Build solutions to the recurrence from the bottom up
  - Identify subproblems
  - Choose a data structure to memoize intermediate results
  - Identify dependencies between subproblems
  - Find an evaluation order so that each subproblem comes after the subproblems it depends on.
  - Implement the algorithm for the evaluation order

Edit Distance

Definitions

- **Edit Distance** or **Levenshtein Distance** is the minimum number of letter insertions, deletions and substitutions required to transform one word into another.
- Example FOOD → MOOD → MOND → MONED → MONEY
- A better way of displaying the transformation is in the next diagram



- **Exercise** find two more 4 step transformations. Are there more ?

Edit Distance

Example 1

- Two further transformations of 4 step transformations of FOOD into MONEY are

$$
\begin{array}{ccccc}
\mathtt{F} & \mathtt{O} & \llcorner\!\lrcorner & \mathtt{O} & \mathtt{D} \\
\updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\
\mathtt{M} & \mathtt{O} & \mathtt{N} & \mathtt{E} & \mathtt{Y}
\end{array}
$$

$$
\begin{array}{ccccc}
\mathtt{F} & \mathtt{O} & \mathtt{O} & \mathtt{D} & \llcorner\!\lrcorner \\
\updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\
\mathtt{M} & \mathtt{O} & \mathtt{N} & \mathtt{E} & \mathtt{Y}
\end{array}
$$

- How do we find such transformations in general ?

Edit Distance

Recursive Algorithm

- **Notation**
- s and t are names for the *start* and *target* strings
- Let $s(i), t(j)$ denote the *prefixes* of s and t of lengths i and j
- Let $s[i], t[j]$ denote the letters at index i and j of s and t — remember that Python and Haskell index from 0 not 1
- Let $e(i,j)$ denote the edit distance between prefixes $s(i)$ and $t(j)$
- We now describe recursively how to transform $s(i)$ to $t(j)$

Edit Distance

Table Dependencies

$$
\begin{tikzcd}
e(i-1, j-1) \arrow[dr, "\text{Match}" above, "\text{Replace}" below] & e(i, j-1) \arrow[d, "\text{Insert}"] \\
e(i-1, j) \arrow[r, "\text{Delete}" below] & e(i, j)
\end{tikzcd}
$$

- **Deletion** — transform $s(i-1)$ to $t(j)$ and delete the last character of $s(i)$ which is $s[i-1]$
- **Insertion** — transform $s(i)$ to $t(j-1)$ and insert the next character required for $t(j)$ which is $t[j-1]$
- **Match/Replace** — transform $s(i-1)$ to $t(j-1)$ and match or replace the last character of $s(i)$ with the last character of $t(j)$ that is, $s[i-1]$ with $t[j-1]$
- The edit distance will be the minimum of the three possibilities
- This also determines the predecessor nodes (1 to 3)

Edit Distance

Base Cases & Table Display

- The shortest way to convert $s(i)$ to an empty string ϵ is by i deletions
- The shortest way to convert an empty string ϵ to $t(j)$ is by j insertions
- **Table Display** — note that in the Haskell (but not the Python), the edit distance table is calculated with (i, j) indexing rows and columns but table is displayed with (j, i) indexing rows and columns
- That means that in the table, the row characters are the source and the column characters are the target, while in the display the column characters are the source.
- This seems to be the convention is most texts but may lead to some tricky thinking when formatting the diagrams
- **Note** The next version will change this design decision.

Edit Distance

Recursive Definition

$$e(i,j) = \begin{cases} i & \text{if } j = 0 \\ j & \text{if } i = 0 \\ \min \left\{ \begin{array}{l} e(i-1,j)+1 \\ e(i,j-1)+1 \\ e(i-1,j-1)+ \\ \quad \text{if } s[i-1] \neq t[j-1] \text{ then } 1 \text{ else } 0 \end{array} \right\} & \text{otherwise} \end{cases}$$

- For simplicity we have assumed the cost of deletion, insertion, or replacement is 1 and the cost of a match is 0 (made more general in the implementation below)
- the running time of the algorithm is exponential in the length of s and t — but we do not care since are going to implement it bottom up.
- The implementation below also provides the graph of the transformations by calculating the predecessors to each node in the table.

Edit Distance

Memoization Table for `editDistance "FOOD" "MONEY"`

	ϵ	F	O	O	D
ϵ	0	1	2	3	4
M	1	1	2	3	4
O	2	2	1	2	3
N	3	3	2	2	3
E	4	4	3	3	3
Y	5	5	4	4	4

Edit Distance

Dynamic Programming Implementation

- If we calculate the edit distance table in the standard row-major order — row by row, each row from left to right
- then when we reach an entry, the entries it depends on are already available
- We develop the algorithm below in both Haskell and Python
- We have used *list comprehensions* in both for iterations — so they should look fairly similar
- *Note* both implementations could be optimised further
- *Health Warning* you are not expected to write the code for this problem — see the comments on `Python_activity_5.11.py` — this is more of a reading/comprehension exercise

Edit Distance

Haskell Implementation (1)

```haskell
type Dist = Int
type Pred = (Int,Int)
type EditDistCell = (Dist, [Pred])
type EditDistTable = [[EditDistCell]]
```

- The type declarations introduce some type aliases

Edit Distance

Haskell Implementation — Service Functions (1)

```haskell
edCellDist :: EditDistCell -> Dist
edCellDist  (x, ps) = x

edCellPreds :: EditDistCell -> [Pred]
edCellPreds (x, ps) = ps
```

- `edCellDist` and `edCellPreds` take apart an `EditDistCell`

Edit Distance

Haskell Implementation — Service Functions (2)

```
getEdCell :: EditDistTable -> (Int,Int) -> EditDistCell
getEdCell edTbl (i,j) = edTbl!!i!!j

getEdDist :: EditDistTable -> (Int,Int) -> Dist
getEdDist  edTbl (i,j)
  = edCellDist  (getEdCell edTbl (i,j))

getEdPreds :: EditDistTable -> (Int,Int) -> [Pred]
getEdPreds edTbl (i,j)
  = edCellPreds (getEdCell edTbl (i,j))
```

Edit Distance

Haskell Implementation — Costs

```haskell
-- Cost of deletion, insertion, substitution
delC, insC, subC :: Dist
delC = 1
insC = 1
subC = 1
```

- The cost of deletion, insertion and replacement are defined here
- The cost of a match is assumed to be 0

Edit Distance

Haskell Implementation — Algorithm

```haskell
editDistTblDP :: String -> String -> EditDistTable
editDistTblDP s t
  = edTbl
    where
    edTbl = [ [ edTblCell (i,j) | j <- [0..length t] ]
              | i <- [0..length s] ]
    edTblCell (0,0) = (0,[])
    edTblCell (i,0) = (i * delC, [(i-1,0)])
    edTblCell (0,j) = (j * insC, [(0,j-1)])
    edTblCell (i,j)
      = (minD, preds)
        where
        possPreds = [(delD, (i-1,j))
                    ,(insD, (i,j-1))
                    ,(subD, (i-1,j-1))
                    ]
        delD = getEdDist edTbl (i-1,j) + delC
        insD = getEdDist edTbl (i,j-1) + insC
        subD = getEdDist edTbl (i-1,j-1)
                 + (if s!!(i-1) == t!!(j-1) then 0 else subC)
        minD = minimum [delD, insD, subD]
        preds = [(i,j) | (x,(i,j)) <- possPreds, x == minD]
```

Edit Distance

Haskell Implementation — Examples

```haskell
edTblTF = editDistTblDP "TREES" "FOREST"
edTblTrTF
  = transpose (editDistTblDP "TREES" "FOREST")

edTblAA = editDistTblDP "ALGORITHM" "ALTRUISTIC"
edTblTrAA
  = transpose (editDistTblDP "ALGORITHM" "ALTRUISTIC")

edTblFM = editDistTblDP "FOOD" "MONEY"
edTblTrFM
  = transpose (editDistTblDP "FOOD" "MONEY")
```

- We use `transpose` since the edit distance table is displayed with the characters of the start string in columns and the target string in the rows with *i* indexing the columns and *j* the rows
- This may lead to some tricky thinking when formatting the diagram but it seems to be the convention.

Edit Distance

Haskell Implementation (2a)

```
editDistTblDP01 :: String -> String -> EditDistTable
editDistTblDP01 s t
  = edTbl
    where
    edTbl = [ [ setEdTblCell (edTbl,s,t) (i,j)
              | j <- [0..length t] ]
              | i <- [0..length s] ]
```

- Defining a subsidiary function `setEdTblCell` avoids having nested `where` clauses
- `setEdTblCell` takes the edit distance table `edTbl`, the start and target strings, and a pair of table indices and returns the table cell.
- The list comprehension definition here is recursive and works because the entries are calculated in row-major order.

Edit Distance

Haskell Implementation (2b)

```haskell
setEdTblCell :: (EditDistTable, String, String)
                -> (Int, Int) -> EditDistCell
setEdTblCell (edTbl,s,t) (0,0) = (0,[])
setEdTblCell (edTbl,s,t) (i,0) = (i * delC, [(i-1,0)])
setEdTblCell (edTbl,s,t) (0,j) = (j * insC, [(0,j-1)])
setEdTblCell (edTbl,s,t) (i,j)
  = (minD, preds)
    where
    possPreds = [(delD, (i-1,j))
                ,(insD, (i,j-1))
                ,(subD, (i-1,j-1))
                ]
    delD = getEdDist edTbl (i-1,j) + delC
    insD = getEdDist edTbl (i,j-1) + insC
    subD = getEdDist edTbl (i-1,j-1)
           + (if s!!(i-1) == t!!(j-1) then 0 else subC)
    minD = minimum [delD, insD, subD]
    preds = [(i,j) | (x,(i,j)) <- possPreds, x == minD]
```

Edit Distance

Haskell Implementation (2c)

```haskell
edTblTF01 = editDistTblDP01 "TREES" "FOREST"
edTblTrTF01
  = transpose (editDistTblDP01 "TREES" "FOREST")

edTblAA01 = editDistTblDP "ALGORITHM" "ALTRUISTIC"
edTblTrAA01
  = transpose (editDistTblDP01 "ALGORITHM" "ALTRUISTIC")

edTblFM01 = editDistTblDP "FOOD" "MONEY"
edTblTrFM01
  = transpose (editDistTblDP01 "FOOD" "MONEY")
```

- `transpose` is used to switch the rows and columns for display
- This may lead to some tricky thinking about formatting the diagram

Edit Distance

Python Implementation

- The Python file `Python_activity_5.11.py` for Python activity 5.11 contains the code for the Dynamic Programming solution for the Levenshtein Edit Distance problem.
- It uses a double `for` loop to do the equivalent calculations of the edit distance table
- TODO rewrite using Python list comprehensions and compare with the Haskell version
- *Note* that the bulk of the code in 5.11 is to format the output and display one path.
- The Haskell to generate the edit table diagrams in these notes is available but not given here since it generates LaTeX TikZ/PGF which is not part of this course.

Edit Distance

Examples

- For `editDistance "ALGORITHM" "ALTRUISTIC"`
  - What is the edit distance ?
  - How many different routes are there ?
  - Give two examples laid out as in the `editDistance "FOOD" "MONEY"` example
- repeat the above with `editDistance "TREES" "FOREST"` (harder!)

Edit Distance

Memoization Table for `editDistance "ALGORITHM" "ALTRUISTIC"`

	ϵ	A	L	G	O	R	I	T	H	M
ϵ	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

Edit Distance

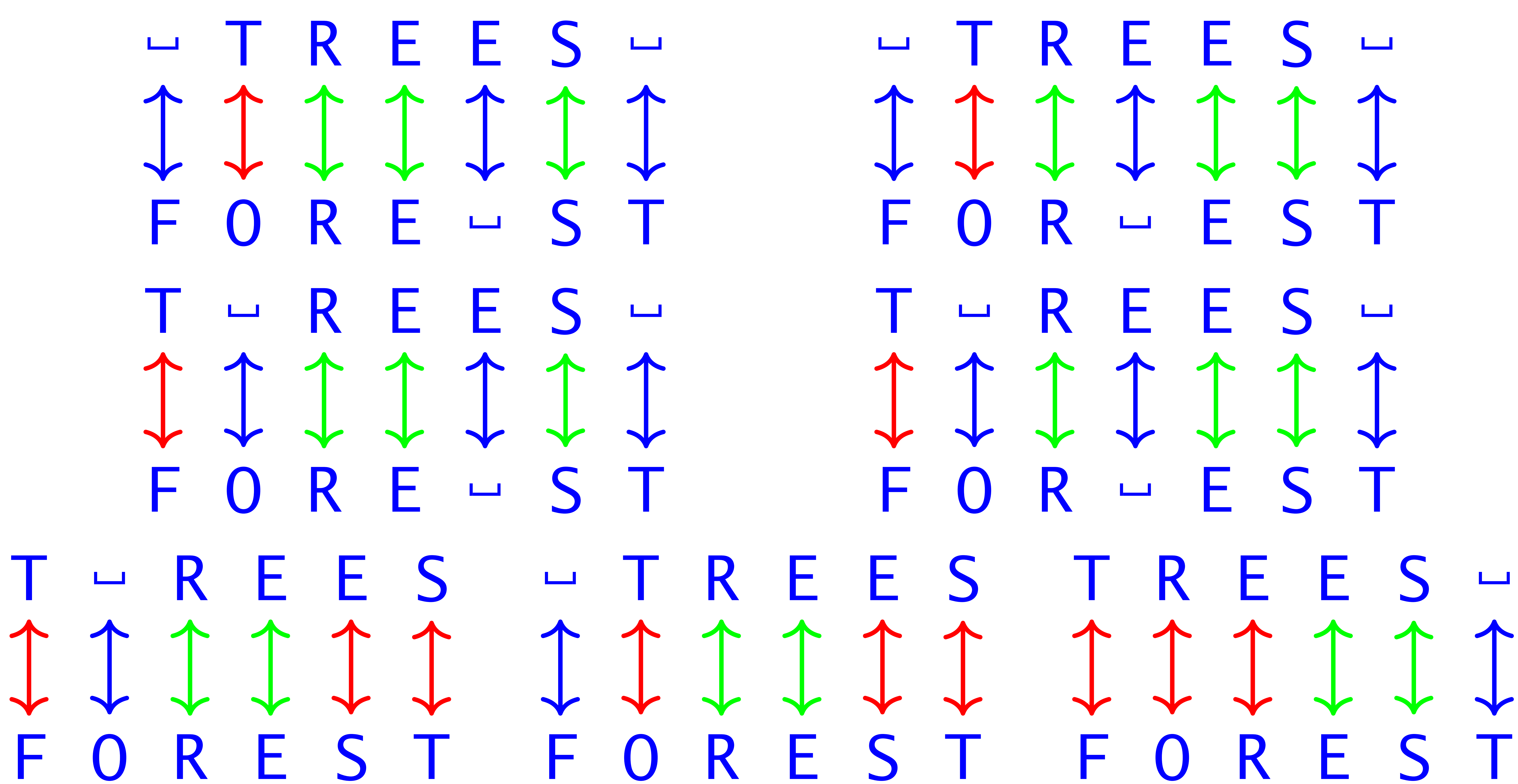
Memoization Table for `editDistance "TREES" "FOREST"`

	ϵ	T	R	E	E	S
ϵ	0	1	2	3	4	5
F	1	1	2	3	4	5
O	2	2	2	3	4	5
R	3	3	2	3	4	5
E	4	4	3	2	3	4
S	5	5	4	3	3	3
T	6	5	5	4	4	4

Edit Distance

Transformation Operations for `editDistance "TREES" "FOREST"`

- Alternative ways of representing the collection of transformation operations were given on slide 121
- Here is a representation of the edit distance graph with match, delete/insert, and replace color arrows



Edit Distance

Sample Transformations for `editDistance "TREES" "FOREST"`

- The transformations can also be represented as a sequence of operations
- Note that the collection of operations could be used in any order — the end result is the same

$$\begin{array}{c}
\text{TREES} \\
\downarrow \text{insert} \\
\text{FTREES} \\
\downarrow \text{replace} \\
\text{FOREES} \\
\downarrow \text{delete} \\
\text{FORES} \\
\downarrow \text{insert} \\
\text{FOREST}
\end{array}
\qquad\qquad
\begin{array}{c}
\text{TREES} \\
\downarrow \text{delete} \\
\text{TRES} \\
\downarrow \text{replace} \\
\text{ORES} \\
\downarrow \text{insert} \\
\text{FORES} \\
\downarrow \text{insert} \\
\text{FOREST}
\end{array}$$

Edit Distance
Sources

- Jeff Erickson Algorithms — these notes are based on the materials on this site
- Wikibooks Levenshtein
- Wikipedia: Levenshtein
- Linux Magazine March 2016 Issue 184 Better Finds — article on egrep which uses the Levenshtein algorithm
- Rosetta Code: Levenshtein Distance many implementations
- Peter Norvig How to Write a Spelling Corrector
- Stack Overflow: Edit Distance in Haskell
- *Levenshtein Algorithm* www.levenshtein.net
- *Levenshtein Demo* odur.let.rug.nl/~kleiweg/lev/
- Wikipedia: Damerau-Levenshtein distance allows for transposition of two adjacent characters

Edit Distance

Diagram Construction

- `editDistTblDP` takes two strings and returns an `EditDistTable`
- The diagrams illustrate `EditDistTable` with the cells laid out in a table with arrows
- The diagrams use the markup language LaTeX with the PGF/TikZ package
- Note: the Haskell to LaTeX and TikZ/PGF diagram code is not part of M269 and is only here for interest
- Here are several small examples

Edit Distance

Memoization Table for `editDistTblDP "ON" "NO"`

	ϵ	O	N
ϵ	0	1	2
N	1	1	1
O	2	1	2

Edit Distance

Memoization Table for `editDistTblDP "OH" "NO"`

	ϵ	O	H
ϵ	0	1	2
N	1	1	2
O	2	1	2

Edit Distance

Memoization Table for `editDistTblDP "IN" "OUT"`

	ϵ	I	N
ϵ	0	1	2
O	1	1	2
U	2	2	2
T	3	3	3

Edit Distance

LaTeX Code for editDistTblDP "OH" "NO"

- The LaTeX code is in a tikzpicture environment
- LaTeX TikZ style definitions
- Styles for nodes and arrows
- Edit path nodes and arrows are red and thick
- Free substitutions (matches) have bold nodes and ultra thick arrows

```
  [ePath/.style={red,thick}
  ,frSub/.style={font=\bfseries}
  ,efrSb/.style={ePath,frSub}
  ,orArr/.style={-Latex}
  ,frArr/.style={orArr,ultra thick}
  ,eoArr/.style={ePath,orArr}
  ,efArr/.style={ePath,frArr}
  ]
```

Edit Distance

LaTeX Code for editDistTblDP "OH" "NO"

- TikZ Matrix code
- The double rows/columns is a hack to ease the frame placement
- See discussion at end

```
% TikZ Matrix code
\matrix (edMat) [matrix of nodes
                ,nodes in empty cells
                ,ampersand replacement=\&
                ,nodes={minimum size=\STtextNodeWidth}]
{
  \&              \&  \&              \&  \&              \&  \&              \& \\
  \&              \&  \&    $\epsilon$ \&  \&            O \&  \&            H \& \\
  \&              \&  \&              \&  \&              \&  \&              \& \\
  \&   $\epsilon$ \&  \&            0 \&  \&            1 \&  \&            2 \& \\
  \&              \&  \&              \&  \&              \&  \&              \& \\
  \&            N \&  \&            1 \&  \&            1 \&  \&            2 \& \\
  \&              \&  \&              \&  \&              \&  \&              \& \\
  \&            O \&  \&            2 \&  \&            1 \&  \&            2 \& \\
  \&              \&  \&              \&  \&              \&  \&              \& \\
};
```

Edit Distance

LaTeX Code for `editDistTblDP "OH" "NO"`

- LaTeX TikZ Arrows code
- All arrows are given the `orArr` style
- TODO: modify the data structure and code to add the edit paths and free substitutions

```
% TikZ Arrows code
\draw[orArr] (edMat-4-4) -- (edMat-4-6);
\draw[orArr] (edMat-4-6) -- (edMat-4-8);

\draw[orArr] (edMat-4-4) -- (edMat-6-4);
\draw[orArr] (edMat-4-4) -- (edMat-6-6);
\draw[orArr] (edMat-6-6) -- (edMat-6-8);
\draw[orArr] (edMat-4-6) -- (edMat-6-8);

\draw[orArr] (edMat-6-4) -- (edMat-8-4);
\draw[orArr] (edMat-6-4) -- (edMat-8-6);
\draw[orArr] (edMat-8-6) -- (edMat-8-8);
\draw[orArr] (edMat-6-6) -- (edMat-8-8);
```

Edit Distance

LaTeX Code for editDistTblDP "OH" "NO"

- LaTeX TikZ Frame code

```
% TikZ frame code
\draw (edMat-1-1.center) -- (edMat-1-9.center)
       -- (edMat-9-9.center) -- (edMat-9-1.center)
       -- cycle;
\draw (edMat-3-1.center) -- (edMat-3-9.center);
\draw (edMat-1-3.center) -- (edMat-9-3.center);
```

Edit Distance

Generating PGF/TikZ from the Edit Table

- We generate the PGF/TikZ from `editDistTblDP`
- By convention we display the transpose of the table
- Remember that the transpose edit cells have lists of predecessors indexed in the original table
- This makes the computation more awkward but it appears to be the convention
- This computation does not include the edit paths nor the free substitutions
- That would be better included in the edit table computation
- TODO: Include edit paths and free substitutions in the edit table computation

Generating PGF/TikZ from the Edit Table

Offsets, Scales and Constants

```
iOffset = 4  -- offset for original string
jOffset = 4  -- offset for target string
iScale  = 2  -- scale for original string
jScale  = 2  -- scale for target string

spc = ' '  -- space char
nl  = '\n' -- new line char
```

Generating PGF/TikZ from the Edit Table

TikZ Arrow Code (1a)

- The code for drawing arrows between nodes is built on the code for one arrow.
- `edCellTrPredToArrow` takes a pair of style name and matrix name,
- a pair of offset and scale data for original and target string,
- the coordinates of the destination node and a predecessor
- and returns the TikZ code for the arrow

Generating PGF/TikZ from the Edit Table

TikZ Arrow Code (1b)

```
edCellTrPredToArrow (styleName,matName)
  ((iOff,iScl),(jOff,jScl)) (j,i) (a,b)
 = "\\draw" ++ "[" ++ styleName ++ "]" ++ [spc]
   ++ "(" ++ matName ++ "-"
   ++ show (jOff + jScl*b)
     ++ "-" ++ show (iOff + iScl*a) ++ ")"
   ++ "␣--␣"
   ++ "(" ++ matName ++ "-"
   ++ show (jOff + jScl*j)
     ++ "-" ++ show (iOff + iScl*i) ++ ");"
   ++ "\n"
```

Generating PGF/TikZ from the Edit Table

TikZ Arrow Code (2a)

- We use `edCellTrPredToArrow` to build a function to take a transposed edit table and return all the arrows as a TikZ string
- `edCellTrPredToArrow01` is a *helper* function to avoid repeat typing
- `edCellTrPredsToArrows01` takes the list of predecessors in a cell and returns the arrows (as a string)
- `edCellTrToArrows01` takes a cell and returns the arrows
- `edRowTrToArrows01` takes a row and returns the arrows
- `edTblTrToArrows01` takes the complete table and returns the arrows

Generating PGF/TikZ from the Edit Table

TikZ Arrow Code (2b)

```
edCellTrPredToArrow01
 = edCellTrPredToArrow
     ("orArr","edMat")
     ((iOffset,iScale),(jOffset,jScale))

edCellTrPredsToArrows01  (j,i) ps
 = concat (map (edCellTrPredToArrow01 (j,i)) ps)

edCellTrToArrows01  (j,i) (d,ps)
 = edCellTrPredsToArrows01 (j,i) ps

edRowTrToArrows01 j cs
 = concat [edCellTrToArrows01 (j,i) c
          | (i,c) <- zip [0..length cs - 1] cs]
          ++ "\n"

edTblTrToArrows01 edTblTr
 = concat [edRowTrToArrows01 j cs
          | (j,cs) <- zip [0..length edTblTr - 1]
                              edTblTr]
```

Generating PGF/TikZ from the Edit Table

TikZ Arrow Code (2c)

```
-- Test Edit Table to Arrows
edTblTrToArrows01TF
 = putStr (edTblTrToArrows01 edTblTrTF)

edTblTrToArrows01AA
 = putStr (edTblTrToArrows01 edTblTrAA)

edTblTrToArrows01FM
 = putStr (edTblTrToArrows01 edTblTrFM)
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (1)

- `edCellTrDistToNode` takes a distance and returns a node string
- `strToNode` takes a string and returns a node string
- **Health Warning** the code below needs rewriting to make it easier to read

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (1a)

```
ndWidth = 13 -- string width of a node
-- nWidth = (length zStr + 1) + 2
-- spc = ' ' -- spc is defined above

ampRepStr = "\\&"

zStr = "$\\epsilon$"

matrixPreamble nm ampRepStr
 = "\\matrix␣(" ++ nm ++ ")␣" ++ "[matrix␣of␣nodes"
   ++ "\n" ++ nSpcs
   ++ ",nodes␣in␣empty␣cells"
   ++ "\n" ++ nSpcs
   ++ ",ampersand␣replacement=" ++ ampRepStr
   ++ "\n" ++ nSpcs
   ++ ",nodes={minimum␣size=\\STtextNodeWidth}]"
   ++ "\n"
   where
   nSpcs = replicate (11 + length nm) spc
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (1b)

```haskell
-- PGF/TikZ Matrix functions

edCellTrDistToNode :: Int -> String
edCellTrDistToNode d
 = replicate n spc ++ dStr ++ [spc]
   where
   dStr = show d
   n    = ndWidth - 1 - length dStr

strToNode str
 = replicate n spc ++ str ++ [spc]
   where
   n = ndWidth - 1 - length str
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (2a)

```
strToPrefixRow str
 = "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ concat ["␣␣" ++ ampRepStr
               ++ replicate ndWidth spc ++ ampRepStr
              | c <- str]
   ++ "␣\\\\\n"
   ++ "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ "␣␣" ++ ampRepStr
   ++ replicate (ndWidth - 11) spc
      ++ "$\\epsilon$␣"++ ampRepStr
   ++ concat ["␣␣" ++ ampRepStr
               ++ replicate (ndWidth - 2) spc
                  ++ [c] ++ [spc] ++ ampRepStr
              | c <- str]
   ++ "␣\\\\\n"
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (2b)

```
strToSuffixRow str
 = "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ concat ["␣␣" ++ ampRepStr
               ++ replicate ndWidth spc ++ ampRepStr
              | c <- str]
   ++ "␣\\\\\n"

colNoTrToPrefixCol zStr str j
 = "␣␣" ++ ampRepStr ++
     (if j == 0
     then replicate (ndWidth - 1 - length zStr) spc
           ++ zStr ++ [spc]
     else replicate (ndWidth - 2) spc
          ++ [str!!(j - 1)] ++ [spc])
   ++ ampRepStr
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (2c)

```haskell
edCellTrToNodes (d,ps)
 = "␣␣" ++ ampRepStr
   ++ edCellTrDistToNode d ++ ampRepStr

edRowTrToNodes zStr str j cs
 = "␣␣" ++ ampRepStr
   ++ replicate ndWidth spc ++ ampRepStr
   ++ concat ["␣␣" ++ ampRepStr
               ++ replicate ndWidth spc ++ ampRepStr
              | c <- cs]
   ++ "␣\\\\\n"
   ++ colNoTrToPrefixCol zStr str j
   ++ concat [edCellTrToNodes c | c <- cs]
   ++ "␣\\\\\n"

edTblTrToNodes zStr str edTblTr
 = concat [edRowTrToNodes zStr str j cs
          | (j,cs) <- zip [0..length edTblTr - 1]
                          edTblTr]
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (2cd)

```
-- Test usage
edTblTrToNodesTF
 = putStr (edTblTrToNodes zStr "FOREST" edTblTrTF)
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (3a)

```
edTblTrToMatrix nm zStr s t
 = matrixPreamble nm ampRepStr
   ++ "{\n"
   ++ strToPrefixRow s
   ++ edTblTrToNodes zStr t edTblTr
   ++ strToSuffixRow s
   ++ "};\n"
   where
   edTblTr = transpose (editDistTblDP s t)
```

Generating PGF/TikZ from the Edit Table

TikZ Matrix Code (3b)

```
-- Test code for entire matrix
edTblTrToMatrixTF
 = putStr (edTblTrToMatrix "edMat" zStr "TREES" "FOREST")

edTblTrToMatrixAA
 = putStr (edTblTrToMatrix "edMat" zStr "ALGORITHM" "ALTRUISTIC")

edTblTrToMatrixFM
 = putStr (edTblTrToMatrix "edMat" zStr "FOOD" "MONEY")
```

Generating PGF/TikZ from the Edit Table

TikZ Picture Code (1)

```
-- Construction of tikzpicture

tikzPreamble
 = "\\begin{tikzpicture}" ++ [nl]
   ++ "␣␣[ePath/.style={red,thick}" ++ [nl]
   ++ "␣␣,frSub/.style={font=\\bfseries}" ++ [nl]
   ++ "␣␣,efrSb/.style={ePath,frSub}" ++ [nl]
   ++ "␣␣,orArr/.style={-Latex}" ++ [nl]
   ++ "␣␣,frArr/.style={orArr,ultra␣thick}" ++ [nl]
   ++ "␣␣,eoArr/.style={ePath,orArr}" ++ [nl]
   ++ "␣␣,efArr/.style={ePath,frArr}" ++ [nl]
   ++ "␣␣]"

tikzEnd
 = "\\end{tikzpicture}" ++ [nl]
```

Generating PGF/TikZ from the Edit Table

TikZ Picture Code (2)

```
tikzFrmCoord nm loc i j
 = "(" ++ nm ++ "-" ++ show j ++ "-"
     ++ show i ++ "." ++ loc ++ ")"

tikzFrame nm loc s t
 = "\\draw␣" ++ (tikzFrmCoord nm loc 1 1) ++ "␣--␣"
       ++ (tikzFrmCoord nm loc xMax 1)
       ++ [nl] ++ dSpcs ++ "␣--␣"
       ++ (tikzFrmCoord nm loc xMax yMax) ++ "␣--␣"
       ++ (tikzFrmCoord nm loc 1 yMax)
       ++ [nl] ++ dSpcs ++ "␣--␣cycle;" ++ [nl]
     ++ "\\draw␣" ++ (tikzFrmCoord nm loc 1 (jScale + 1))
       ++ "␣--␣" ++ (tikzFrmCoord nm loc xMax (jScale + 1))
       ++ ";" ++ [nl]
     ++ "\\draw␣" ++ (tikzFrmCoord nm loc (iScale + 1) 1)
       ++ "␣--␣" ++ (tikzFrmCoord nm loc (iScale + 1) yMax)
       ++ ";" ++ [nl]
     where
       xMax = iOffset + 1 + iScale * (length s)
       yMax = jOffset + 1 + jScale * (length t)
       dSpcs = replicate 6 spc
```

Generating PGF/TikZ from the Edit Table

TikZ Picture Code (3)

```
edTblTrToTikz nm loc zStr s t
 = tikzPreamble ++ [nl]
   ++ [nl] ++ "%␣TikZ␣Matrix␣code" ++ [nl]
   ++ edTblTrToMatrix nm zStr s t ++ [nl]
   ++ [nl] ++ "%␣TikZ␣Arrows␣code" ++ [nl]
   ++ edTblTrToArrows01 edTblTr ++ [nl]
   ++ [nl] ++ "%␣TikZ␣frame␣code" ++ [nl]
   ++ tikzFrame nm loc s t ++ [nl]
   ++ tikzEnd
   where
     edTblTr = transpose (editDistTblDP s t)
```

Generating PGF/TikZ from the Edit Table

Test Code Matrix Frame

```haskell
-- Test code for matrix frame lines

tikzFrameFM
 = putStr (tikzFrame "edMat" "center"
             "FOOD" "MONEY")

tikzFrameAA
 = putStr (tikzFrame "edMat" "center"
             "ALGORITHM" "ALTRUISTIC")

tikzFrameTF
 = putStr (tikzFrame "edMat" "center"
             "TREES" "FOREST")
```

Generating PGF/TikZ from the Edit Table

Test Code TikZ Picture

```
-- Text code for tikzpicture

edTblTrToTikzFM
 = putStr (edTblTrToTikz "edMat" "center" zStr
           "FOOD" "MONEY")

edTblTrToTikzAA
 = putStr (edTblTrToTikz "edMat" "center" zStr
           "ALGORITHM" "ALTRUISTIC")

edTblTrToTikzTF
 = putStr (edTblTrToTikz "edMat" "center" zStr
           "TREES" "FOREST")

edTblTrToTikzKK
 = putStr (edTblTrToTikz "edMat" "center" zStr
           "KITTEN" "KNITTING")

edTblTrToTikzSE
 = putStr (edTblTrToTikz "edMat" "center" zStr
           "SIX" "ELEVEN")
```

Edit Distance

Memoization Table for `editDistance "KITTEN" "KNITTING"`

	ϵ	K	I	T	T	E	N
ϵ	0	1	2	3	4	5	6
K	1	0	1	2	3	4	5
N	2	1	1	2	3	4	4
I	3	2	1	2	3	4	5
T	4	3	2	1	2	3	4
T	5	4	3	2	1	2	3
I	6	5	4	3	2	2	3
N	7	6	5	4	3	3	2
G	8	7	6	5	4	4	3

Edit Distance

Memoization Table for `editDistance "SIX" "ELEVEN"`

	ϵ	S	I	X
ϵ	0	1	2	3
E	1	1	2	3
L	2	2	2	3
E	3	3	3	3
V	4	4	4	4
E	5	5	5	5
N	6	6	6	6

M269 Library

Overview

- Implementations in Python of some M269 algorithms and data structures
- M269 Library contains
  - `bst.py` — Binary Search Tree
  - `deque.py` — Double-ended Queue
  - `digraph.py` — Directed graph
  - `graph.py` — Graph (discrete mathematics)
  - `hash_table.py` — Hash table
  - `priority_queue.py` — Priority queue
  - `queue.py` — Queue (abstract data type)
  - `sort.py` — Sorting algorithm
  - `stack.py` — Stack (abstract data type)

Future Work

Topics & Events

- Saturday, 13 March 2021 tutorial online
- Wednesday, 24 March 2021 TMA02 due
- Sunday, 11 April 2021 online tutorial Unit 6 Logic
- Wednesday 28 April 2021 iCMA46 due
- Sunday, 2 May 2021 online tutorial Unit 7 Computability, Complexity
- Sunday, 16 May 2021 online tutorial exam revision
- Saturday, 22 May 2021 tutorial online
- Tuesday 25 May 2021 iCMA47 due
- Tuesday 8 June 2022 Exam
- Please email me with any requests for particular topics