M269 Exams

Prsntn 2017J Exam

Phil Molyneux

7 June 2018

M269 2017J Exam
Qs

- M269 Algorithms, Data Structures and Computability
- Presentation 2017J Exam
- Date Thursday, 7 June 2018 Time 10:00–13:00
- There are **TWO** parts to this examination. You should attempt all questions in **both** parts
- **Part 1** carries **65 marks — 80 minutes**
- **Part 2** carries **35 marks — 90 minutes**
- **Note** see the original exam paper for exact wording and formatting — these slides and notes may change some wording and formatting
- **Note** The 2015J exam and before had Part 1 with 60 marks (100 minutes), Part 2 with 40 marks (70 minutes)

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Q Part1

- Answer every question in this part.
- The marks for each question are given below the question number.
- Answers to questions in this Part should be written **on this paper** in the spaces provided, or in the case of multiple-choice questions you should tick the appropriate box(es).
- If you tick **more** boxes than indicated for a multiple choice question, you will receive **no** marks for your answer to that question.
- Use the provided answer books for any rough working.

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Q 1 (2 marks)

- Which **one** of the following statements is true? (Tick **one** box.)

A. An Abstract Data Type is the definition of a data structure in terms of the pre- and postconditions on the data structure.

B. A more complex algorithm will always take more time to execute than a less complex one.

C. Abstraction as modelling involves two layers — the interface and the implementation.

D. A problem is computable if it is possible to build an algorithm which solves any instance of the problem in a finite number of steps.

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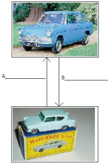
Q 2 (2 marks)

- The general idea of abstraction as modelling can be shown with the following diagram.
- The picture in the top is of a Ford Anglia in the real world, and the picture in the bottom is of a Matchbox model of a Ford Anglia.



- Complete the diagram by adding an appropriate label in the space indicated by **A** and one in the space indicated by **B**.

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Q 3 (4 marks)

- A binary search is being carried out on the list shown below for item 41:
  `[2,16,17,25,31,39,41,52,67,69,77,83,89,91,99]`
- For each pass of the algorithm, draw a box around the items in the partition to be searched during that pass, continuing for as many passes as you think are needed.
- We have done the first pass for you showing that the search starts with the whole list. Draw your boxes below for each pass needed; you may not need to use all the lines below. *(The question had 8 rows)*

(Pass 1) [$\boxed{\texttt{2,16,17,25,31,39,41,52,67,69,77,83,89,91,99}}$]

(Pass 2) `[2,16,17,25,31,39,41,52,67,69,77,83,89,91,99]`

(Pass 3) `[2,16,17,25,31,39,41,52,67,69,77,83,89,91,99]`

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Q 4 (5 marks)

- A Python program contains a loop with the following guard

```
while not (x >= 2 or y <= 2) or (x < 2 and y > 2):
```

- Complete the following truth table, where:

  P represents $x < 2$
  Q represents $y > 2$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$	$P \wedge Q$	$\neg(\neg P \vee \neg Q) \vee (P \wedge Q)$
F	F						
F	T						
T	F						
T	T						

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Q 4 (contd)

- Use the results from your truth table to choose which one of the following expressions could be used as the simplest equivalent to the above guard. (Tick **one** box.)

A. **not** $(x < 2$ **and** $y > 2)$

B. $(x >= 2$ **or** $y <= 2)$

C. $(x < 2$ **and** $y > 2)$

D. $(x >= 2$ **and** $y <= 2)$

E. $(x < 2$ **and** $y <= 2)$

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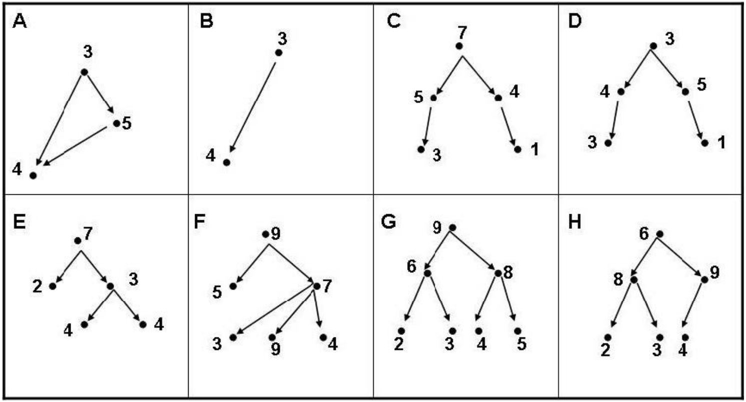
Q 5 (4 marks)

- Consider the diagrams in A–H, where nodes are represented by black dots and edges by arrows. The numbers are the keys for the corresponding nodes.



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Q 5

- On each line, write one or more letters, or write *None*.

(a) Which of **A**, **B**, **C** and **D**, if any, are **not** a tree?

(b) Which of **E**, **F**, **G** and **H**, if any, are binary trees?

(c) Which of **C**, **D**, **G** and **H**, if any, are complete binary trees?

(d) Which of **C**, **D**, **G** and **H**, if any, are not a heap?

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Q 6 (6 marks)

- Consider the following function, which takes a non-empty list as an argument.

```python
def variance(aList):
    n = len(aList)
    total = 0
    for item in aList:
        total = total + item
    mean = total / n
    ssdev = 0
    for item in aList:
        deviation = item - mean
        ssdev = ssdev + (deviation * deviation)
    var = ssdev / n
    return var
```

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Q 6 (contd)

- From the options below, select the two that represent the correct combination of $T(n)$ and Big-O complexity for this function.

  You may assume that a step (i.e. the basic unit of computation) is the assignment statement.

  (Tick **one** box for T(n) and **one** box for Big-O complexity.)

A.	$T(n) = 2n + 5$	i.	$O(n)$
B.	$T(n) = 3n + 5$	ii.	$O(2n)$
C.	$T(n) = 3n + 6$	iii.	$O(3n)$
D.	$T(n) = n^2 + 5$	iv.	$O(n^2)$
E.	$T(n) = 3n^2 + 6$	v.	$O(3n^2)$

- Explain how you arrived at $T(n)$ and the associated Big-O

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Q 7 (4 marks)

(a) Which **one** of the following statements are true? (Tick **one** box.)

A. Hash tables store unique (i.e. non-duplicate) keys in an arbitrary order and are therefore an implementation of the Set ADT.

B. A hash function maps a value to a key in the table.

C. The higher the load factor on a hash table, the higher the risk of collisions.

D. Linear Probing is a chaining technique designed to resolve collisions.

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Q 7 (contd)

(b) Calculate the load factor for the hash table below. Show your working.

0	1	2	3	4	5	6	7	8	9	10	11
	Alice			Nisha	Bob				Ali		

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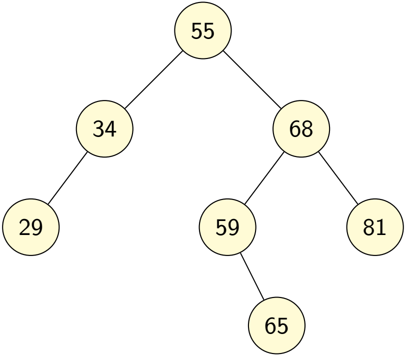
Q 8 (4 marks)

- In the following binary search tree, label each node with its balance factor.



- Would this tree need to be rebalanced to be a valid AVL tree? Explain your answer.

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Q 9 (4 marks)

- A water distribution network can be represented as a weighted directed graph.
- The nodes represent the reservoirs, water treatment centres and consumers (homes, factories, etc.).
- The directed edges represent the water pipes, showing the flow of water, from the reservoirs to the consumers, via the treatment centres.
- The edge weights indicate the maximum flow (in cubic metres per second) of the pipes.
- Complete the following statements, and include in the justification any assumptions you make.
- For a typical water distribution network, the graph is (*choose from CYCLIC/ACYCLIC*) because:
- and it is (*choose from SPARSE/DENSE*) because:

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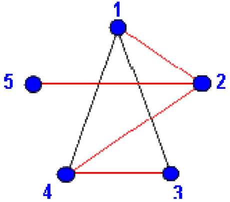
Q 10 (4 marks)

- Consider the following undirected graph:



- Complete the table below to show **one** order in which the vertices of the above graph could be visited in a **Breadth** First Search (BFS) starting at vertex 3:

Vertices visited	3				

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Q 11 (4 marks)

- In propositional logic, what does it mean to say that a well-formed formula is *contingent*?
- Is the well-formed formula $(P \to Q) \to (\neg Q \to \neg P)$ contingent? Explain.

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Q 12 (6 marks)

- Consider the following particular interpretation $\mathcal{I}$ for predicate logic allowing facts to be expressed about people and the computer games they own and play.
- The domain of individuals is $\mathcal{D} = \{$Jane, John, Saira, Gran Turismo, Kessen, Pacman, The Sims, Pop Idol$\}$.
- The constants *jane*, *john*, *saira*, *gran_turismo*, *kessen*, *pacman*, *the_sims* and *pop_idol* are assigned to the corresponding individuals.

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Q 12 (contd)

- Two predicate symbols are assigned binary relations as follows:
- $\mathcal{I}(owns) = \{$(Jane, Gran Turismo), (Jane, Kessen), (John, Pacman), (John, The Sims), (John, Pop Idol), (Saira, Pop Idol), (Saira, Kessen)$\}$
- $\mathcal{I}(has\_played) = \{$(Jane, Gran Turismo), (Jane, Pop Idol), (Jane, Kessen), (John, The Sims), (John, Pop Idol), (Saira, Gran Turismo), (Saira, The Sims)$\}$

- Q 12 continued on next slide

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Q 12 (contd)

(a) Consider the sentence in English: *Jane owns all the games she has played*.

Which **one** of these well-formed formulae is a translation of the sentence into predicate logic?

A. $\forall X.(\mathit{owns}(\mathit{jane}, X) \rightarrow \mathit{has\_played}(\mathit{jane}, X))$
B. $\forall X.(\mathit{has\_played}(\mathit{jane}, X) \rightarrow \mathit{owns}(\mathit{jane}, X))$
C. $\forall X.(\mathit{has\_played}(\mathit{jane}, X) \wedge \mathit{owns}(\mathit{jane}, X))$

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Q 12 (contd)

(b) Give an appropriate translation of the well-formed formula below into English

$\exists X.(\neg owns(saira, X) \land has\_played(jane, X))$

- This formula is (*choose from TRUE/FALSE*), under the interpretation given on the previous page.
- Explain why in the box below.

  You need to consider any relevant values for the variables, and show, using the domain and interpretation on the previous page, whether they make the formula TRUE or FALSE.

  In your explanation, make sure that you use formal notation.

  For example, instead of stating *John doesn't own Kessen* you need to write $(\text{John}, \text{Kessen}) \notin \mathcal{I}(owns)$

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Q 13 (6 marks)

- A database contains the following tables:

oilfield

name	production
Warga	3
Lolli	5
Tolstoi	0.5
Dakhun	2
Sugar	3

operator

company	field
Amarco	Warga
Bratape	Lolli
Rosbif	Tolstoi
Taqar	Dakhun
Bratape	Sugar

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Q 13 (contd)

(a) For the following SQL query, give the table returned by the query.

```sql
SELECT name, company
FROM oilfield CROSS JOIN operator
WHERE name = field ;
```

- Write the question that the above query is answering.
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Q 13 (contd)

(b) Write an SQL query that answers the question

What is the name and the operating company of each oil field operated by Bratape?

Your query should return the following table.

company	**field**
Bratape	Lolli
Bratape	Sugar

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Q 14 (6 marks)

- Consider the following decision problems:

1. The Equivalence Problem
2. Is a given list not empty?
3. The Halting Problem
4. Is a given binary tree balanced?

- On each line, write one or more of the above problem numbers, or write *None*.
- Which problems, if any, are decidable?
- Which problems, if any, are tractable?
- Which problems, if any, are NP-hard?

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Q 15 (4 marks)

- Which **two** of the following statements are true? (Tick **two** boxes.)

A. If a programming language, let's call it PL, is Turing complete, then any computational problem can be solved with a program written in PL.

B. The Equivalence Problem is not computable.

C. Problems in the class NP are defined as problems for which it is not known whether they're tractable.

D. There are non-computable computational problems because: There are more decision problems with the natural numbers as their domain (DPN) than Turing Machines that solve instances of DPN.

E. The Totality Problem is definitely in the class P.

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Q Part2

- Answer every question in this Part.
- Answers to questions in this Part must be written in the separate **answer books**, which you should also use for your rough working.

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Q 16 (20 marks)

- Consider an ADT for **undirected** graphs, named UGraph, that includes these operations:
- `nodes`, which returns a sequence of all nodes in the graph, in no particular order;
- `has_edge`, which takes two nodes and returns true if there is an edge between those nodes;
- `edges`, which returns a sequence of node-node pairs (tuples), in no particular order. Each edge only appears once in the returned sequence, i.e. if the pair (node1, node2) is in the sequence, the pair (node2, node1) is not.
- How each node is represented is irrelevant.
- You can assume the graph is connected and has no edge between a node and itself.
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Q 16 (contd)

(a) The following stand-alone Python function checks if an undirected graph is complete, i.e. if each node is connected to every other node.

It assumes the ADT is implemented as a Python class.

```
def is_complete(graph):
  nodes = graph.nodes()
  for node1 in nodes:
    for node2 in nodes:
      edge_exists = graph.has_edge(node1, node2)
      if node1 != node2 and not edge_exists:
        return False
  return True
```

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Q 16 (contd)

- Assume that `graph.nodes` has complexity $O(n)$, where n is the number of nodes, and `graph.has_edge` has complexity $O(1)$.
- State and justify a bestcase scenario and a worst-case scenario for the above function, and their corresponding Big-O complexities.
- Assume the basic computational step is the assignment.
- State explicitly any other assumptions you make.

(7 marks)

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Q 16 (contd)

(b) In graph theory, the number of nodes in a graph is called the order of the graph.
The term *order* is unrelated to sorting.

(i) Specify the problem of calculating the order of an undirected graph by completing the following template.
Note that it is specified as an independent problem, not as a UGraph operation.
You may write the specification in English and/or formally with mathematical notation. **(4 marks)**

Name: order
Inputs
Preconditions:
Outputs:
Postconditions:

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Q 16 (contd)

(ii) Give your initial insight for an algorithm that solves the problem.

Of the ADT operations given above you may only use `edges`. **(4 marks)**

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Q 16 (contd)

(c) A city council is planning the city's bus routes.

It has decided which places will have a bus stop (schools, cinemas, hospital, etc.).

Each bus route will start from the train station, visit a number of bus stops, and then return through the same streets to the station, visiting the same bus stops in reverse order. Each bus stop has to be served by at least one bus route. The council wants to minimize the total amount of time that all buses are on the road when following their routes.

- State and justify which data structure(s) and algorithm(s) you would adopt or adapt to solve this problem.

  State explicitly any assumptions you make. **(5 marks)**

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Q 17 (15 marks)

- Imagine you are working for a logistics company that currently uses heuristic algorithms to send their trucks on round trips that use as little fuel as possible.
- The morning paper reports that P=NP has been proved through the discovery of a tractable algorithm for the SAT problem.
- What does this news mean for the company?
- Q 17 continued on next slide

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Q 17 (contd)

- Write a brief memo with your advice on this matter to the board of the company, which doesn't include any computing experts.
  The memo must have the following structure:

1. A suitable title.
2. A paragraph *setting the scene* and introducing the key question.
3. A paragraph in which you describe in layperson's terms what P=NP means.
4. A paragraph describing briefly how P=NP may impact on the company's main business objective (the cost-effective use of their trucks).
5. A conclusion on what you propose the company should do in face of this news, if anything.

- Q 17 continued on next slide

Go to Soln 17

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Q 17 (contd)

- Some marks will be awarded for a clear coherent text that is appropriate for its audience, so avoid unexplained technical jargon and abrupt changes of topic, and make sure your sentences fit together to tell an overall *story*.

  As a guide, you should aim to write roughly two to five sentences per paragraph.

Go to Soln 17

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Solns

- The solutions given below are not official solutions
- For some questions, alternatives are given — a student would only have to provide one

Go to Qs

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Soln Part1

- Part 1 solutions

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Soln 1

A. An Abstract Data Type is the definition of a data structure in terms of the pre- and postconditions on the data structure. **No** *ADT defined by operations that may be performed on it and the pre- and postconditions on the operations*

B. A more complex algorithm will always take more time to execute than a less complex one. **No** *The less complex one could have a bigger problem*

C. Abstraction as modelling involves two layers — the interface and the implementation. **No** *Models represent reality in sufficient detail*

D. A problem is computable if it is possible to build an algorithm which solves any instance of the problem in a finite number of steps. **Yes**

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Soln 2

- **A** (Model) ignores detail of
- **B** (Actual car) represented by

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Soln 3

- The complete binary search:

  (Pass 1) [$\boxed{\texttt{2,16,17,25,31,39,41,52,67,69,77,83,89,91,99}}$]

  (Pass 2) [$\boxed{\texttt{2,16,17,25,31,39,41}}$,52,67,69,77,83,89,91,99]

  (Pass 3) [2,16,17,25,$\boxed{\texttt{31,39,41}}$,52,67,69,77,83,89,91,99]

  (Pass 4) [2,16,17,25,31,39,$\boxed{\texttt{41}}$,52,67,69,77,83,89,91,99]

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Soln 4

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$	$P \wedge Q$	$\neg(\neg P \vee \neg Q) \vee (P \wedge Q)$
F	F	T	T	T	F	F	F
F	T	T	F	T	F	F	F
T	F	F	T	T	F	F	F
T	T	F	F	F	T	T	T

- The equivalent expression is C.
- Soln 4 continued on next slide

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Soln 4

A. **not** $(x < 2$ **and** $y > 2)$
  $\rightarrow$ **not** P **and** **not** Q

B. $(x >= 2$ **or** $y <= 2)$
  $\rightarrow$ **not** P **or** **not** Q

C. $(x < 2$ **and** $y > 2)$ $\rightarrow$ P **and** Q

D. $(x >= 2$ **and** $y <= 2)$
  $\rightarrow$ **not** P **and** **not** Q

E. $(x < 2$ **and** $y <= 2)$
  $\rightarrow$ P **and** **not** Q

- **not** (**not** P **or** **not** Q) **or** (P **and** Q)
  $\rightarrow$ $(P$ **and** $Q)$ **or** $(P$ **and** $Q)$
  $\rightarrow$ $(P$ **and** $Q)$

Go to Q 4

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Soln 5

(a) Which of **A**, **B**, **C** and **D**, if any, are **not** a tree?

A is not a tree since 4 has two parents

(b) Which of **E**, **F**, **G** and **H**, if any, are binary trees?

E, **G** and **H** — **F** is not a binary tree since 7 has three sub-trees — note **E** has duplicate nodes

(c) Which of **C**, **D**, **G** and **H**, if any, are complete binary trees?

G and **H** — **E** is not a complete binary tree since the last level is not filled from left to right

(d) Which of **C**, **D**, **G** and **H**, if any, are not a heap?

C (since not a complete binary tree), **D** (since misses both properties), **H** (since does not have ordering property)

Go to Q 5

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Soln 6

- Options **B** and **i**
- There are two loops (not nested) with 3 assignments which contribute $3n$ to $T(n)$
- The remainder of the code has 5 assignments
- Hence $T(n) = 3n + 5$
- and complexity is $O(n)$ from the leading term

Go to Q 6

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Soln 7

A. Hash tables store unique (i.e. non-duplicate) keys in an arbitrary order and are therefore an implementation of the Set ADT. **No** — the order is not arbitrary, it is a result of the hash function and any collision resolution

B. A hash function maps a value to a key in the table. **No** — a hash function maps values to integer indices of a table, but that position may be occupied.

C. The higher the load factor on a hash table, the higher the risk of collisions. **Yes** — a high load factor means a high proportion of the hash table is occupied

D. Linear Probing is a chaining technique designed to resolve collisions. **No** — Linear probing and chaining are different techniques

(b) The load factor is 4/12 or 0.3333

Go to Q 7

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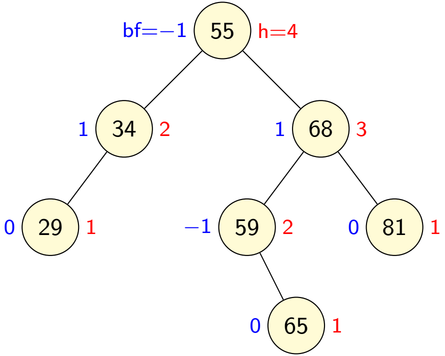
Soln 8

- Binary tree with balance factors and heights — note: here empty trees have height 0 (not -1)



- The tree would not need rebalancing to be an AVL tree — the tree is a binary search tree and every node has balance factor in the range $\{-1, 0, +1\}$

Go to Q 8

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Soln 9

- The network is *acyclic* since water does not return to the sources (in this network) — no mention is made of waste water and sewerage collection and recycling.
- A *sparse* network since most nodes are only connected to one other node.

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Soln 10

- Possible answers:

Vertices visited	3	1	4	2	5

Vertices visited	3	4	1	2	5

Go to Q 10

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Soln 11

- A WFF is *contingent* if it is true in some interpretations and false in others — a *tautology* is true in every interpretation, a *contradiction* is false in every interpretation.
- $(P \to Q) \to (\neg Q \to \neg P)$ is a *tautology*

  $\equiv \neg(\neg P \vee Q) \vee (\neg\neg Q \vee \neg P)$ by rewriting $\to$

  $\equiv \neg(\neg P \vee Q) \vee (\neg P \vee Q)$ by negation and commutativity

  $\equiv$ True by negation

P	Q	$(P \to Q)$	$(\neg Q \to \neg P)$	$(P \to Q) \to (\neg Q \to \neg P)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

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Soln 12

(a) *Jane owns all the games she has played* means

If Jane has played X then Jane owns X

so the answer is

B. $\forall X.(has\_played(jane, X) \rightarrow owns(jane, X))$

- A. $\forall X.(owns(jane, X) \rightarrow has\_played(jane, X))$ means *Jane has played all the games she owns*
- B. $\forall X.(has\_played(jane, X) \wedge owns(jane, X))$ means *Jane owns all games and has played all of them*

- Soln 12 continued on next slide

Go to Q 12

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Soln 12 (contd)

(b) $\exists X.(\neg \mathit{owns}(\mathit{saira}, X) \to \mathit{has\_played}(\mathit{jane}, X))$ means
There is at least one game that Saira does not own that Jane has played

- **True**

  because Jane has played Gran Turismo but Saira does not own it

- $(\text{Saira}, \text{Gran Turismo}) \notin \mathcal{I}(\mathit{owns})$
  $\wedge\ (\text{Jane}, \text{Gran Turismo}) \in \mathcal{I}(\mathit{has\_played})$

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Soln 13

```sql
SELECT name, company
FROM oilfield CROSS JOIN operator
WHERE name = field ;
```

- Table returned by the query

Warga	Amarco
Lolli	Bratape
Tolstoi	Rosbif
Dakhun	Taqar
Sugar	Bratape

- SQL for *What is the name and the operating company of each oil field operated by Bratape?*

```sql
SELECT company, field
FROM operator
WHERE company = 'Bratape'
```

Go to Q 13

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Soln 14

- Decidable: 2. (Empty list), 4. (Balanced binary tree)
- Tractable: 2. (Empty list), 4. (Balanced binary tree)
- NP-hard: 3. (Halting problem)

  See StackOverflow: Proof that the halting problem is NP-hard?

Go to Q 14

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Soln 15

A. **False** PL, Turing complete programming language can compute anything that is computable but there are some computational problems that are not computable
B. **True** Equivalence Problem is not computable — see Computability notes
C. **False** The class P is a subset of NP — we just do not know whether it is a proper subset or equal
D. **True** Programs are finite strings over a finite alphabet (ASCII or Unicode) hence countable — however the number of different languages over any alphabet of more than one symbol is uncountable — a problem is really membership of a string in some language
E. **False** Totality Problem is not computable — see Computability notes — so not in the class P

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Soln Part2

- Part 2 solutions

Go to Q Part2

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Soln 16

(a) Best case: First node in `nodes` has no edge to the second node in `nodes` (the first being itself) — hence returns `False` with only two calls in the inner loop — so $O(n)$

Worst case: The graph is complete and $O(n^2)$ since both loops fully traversed

- Soln 16 continued on next slide

Go to Q 16

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Soln 16 (contd)

(b) **(i)** Specification of `order` function

Name: order
Inputs: undirected graph, `g`
Preconditions: `g` is connected
Outputs: Integer, `n`
Postconditions: `n` is the size of the set of nodes in `g`

- **(ii)** Use `edges` to give a sequence of edges;
  extract a list of the first and second nodes in each edge;
  remove duplicates in the list (making a set);
  the size of the result is the order of the graph (assumes connected graph)

- Soln 16 continued on next slide

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Soln 16 (contd)

(c) Data structures: graph with bus stops as nodes and weighted edges as distance between stops;

- Algorithm(s): Some variant on Prim's algorithm for minimum spanning tree.

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Soln 17

- Follow the given structure:
- Title: *given at the end*
- Setting the scene:
- P as the class of problems with solutions that are found in time which is a fixed polynomial of the input size $O(n^k)$
- NP as the class of problems with solutions that can be checked in polynomial time

- Soln 17 continued on next slide

Go to Q 17

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Soln 17 (contd)

- Give examples of both:
- Pairing problem: given a group of students and knowledge of which are compatible, place them in compatible groups of 2 — Edmonds (1965) showed there is a polynomial time algorithm for this
- Partition into Triangles: make groups of three with each pair in the group compatible
- Find a large group of students who are compatible — Clique problem
- Sit the students round a large table so that no incompatible students are next to each other (Hamiltonian Cycle)
- The first problem is in P, the others are in NP (we can check a solution) but it is not known if they are in P
- Soln 17 continued on next slide

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Soln 17 (contd)

- Define NP complete problems, dp: (a) In NP; (b) Every problem in NP is reducible to dp in polynomial time
- If P=NP then every NP problem would have a polynomial time solution — possibly via reduction to the SAT problem
- However proving P=NP (a) may not actually give an algorithm in polynomial time for solving an NP complete problem (the newspaper says there is a tractable algorithm for SAT) (b) Even with a tractable algorithm for SAT, the $O(n^k)$ may be very large.
- Give example of linear programming: standard simplex algorithm is exponential (worst case) while the ellipsoid algorithm is polynomial — however in practice simplex is used (because it is good enough) (see Wikipedia: LP)
- Soln 17 continued on next slide

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Soln 17 (contd)

- Implications: Good: all optimisation problems become tractable including vehicle routing
- Implications: Bad: Public key cryptography becomes impossible, banking transactions become tricky to carry out securely, the same applies to secure Web transactions
- Conclusion: prepare for huge disruption — this is bigger that the Internet or the Web
- Title: P=NP — a Disruptive Discovery

- Soln 17 continued on next slide

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Soln 17 (contd)

- **Reading**
- StackExchange: What would be the impact of P=NP?
- Lance Fortnow: The Status of the P Versus NP Problem readable article in 2009 CACM
- The International SAT Competitions Web Page
- Lance Fortnow: *The Golden Ticket: P, NP and the Search for the Impossible* (2013,2017)
- Lance Fortnow, Steve Homer: *A Short History of Computational Complexity*
- Computational Complexity blog from Lance Fortnow and Bill Gasarch

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