M269 Exams

Prsntn 2014J Exam

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29 August 2016

M269 2014J Exam
Q s

- M269 Algorithms, Data Structures and Computability
- Presentation 2014J Exam
- Date Monday, 8 June 2015 Time 10:00–13:00
- There are **TWO** parts to this examination. You should attempt all questions in **both** parts
- **Part 1** carries **60 marks — 100 minutes**
- **Part 2** carries **40 marks — 70 minutes**
- **Note** see the original exam paper for exact wording and formatting — these slides and notes may change some wording and formatting

Go to Exam Soln s

M269 2014J Exam
Q Part1

- Answer every question in this part.
- Answers to questions in this part should be written on this paper in the spaces provided, or in the case of multiple-choice questions you should tick the appropriate box(es).
- If you tick more boxes than indicated for a multiple choice question, you will receive **no** marks for your answer to that question.

Go to Exam Soln Part1

M269 2014J Exam
Q 1

- **Question 1** Which **two** of the following statements are true ? **(2 marks)**

A. A computational thinker can explain why some computational problems do not have a good computational solution.

B. The main factor when deciding the best algorithm for a given task is the programming language in which the algorithm is to be implemented.

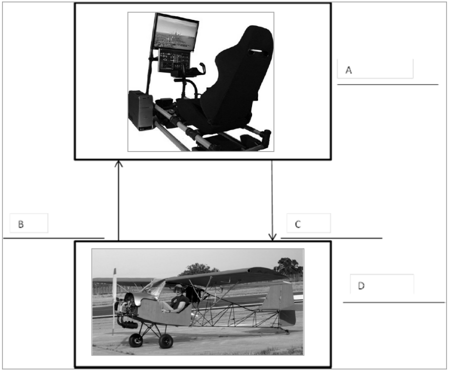
C. A decision problem can be defined as a formally stated problem to which the answer is either yes or no.

D. The data structure chosen to hold data does not affect the efficiency of an algorithm operating on that data.

Go to Exam Soln 1

M269 2014J Exam
Q 2

- **Question 2** The diagram below shows images of a flight simulator for a single seater aircraft and an actual single seater aircraft. **(2 marks)**



- Q 2 continued on next slide

Go to Exam Soln 2

M269 2014J Exam
Q 2

- Complete the diagram above by adding an appropriate label (one of the numbers 1 to 8) in each of the spaces indicated by A, B, C and D. The possible answers are shown as 1 to 8 below.

1 ... represented by

2 ... ignores detail of

3 ... solves

4 ... transforms

5 ... model

6 ... part of reality

7 ... computational problem

8 ... layer

Go to Exam Soln 2

M269 2014J Exam

Q 3

- **Question 3** An insertion sort is being carried out on the list of integers shown below, so as to arrange the list in ascending numerical order: **(4 marks)**

start array	54	26	93	17	77	44	55	31

- For the first four passes of the algorithm (after assuming that a list with one item is already sorted), show the order of the items in the list after that pass:

(Pass 1)								
(Pass 2)								
(Pass 3)								
(Pass 4)								

Go to Exam Soln 3

M269 2014J Exam

Q 4

- **Question 4** Consider the guard in the following Python `while` loop header: **(4 marks)**

```
while (b > 8) and not(a < 6 or b <= 8):
```

(a) Make the following substitutions:

P represents `a < 6`

Q represents `b > 8`

complete the following truth table:

P	Q	$\neg Q$	$P \vee \neg Q$	$\neg(P \vee \neg Q)$	$Q \wedge \neg(P \vee \neg Q)$
T	T				
T	F				
F	T				
F	F				

- Q 4 continued on next slide

Go to Exam Soln 4

M269 2014J Exam
Q 4 (contd)

(b) Use the results from your truth table to choose which one of the following expressions could be used as a simpler equivalent to the above guard.

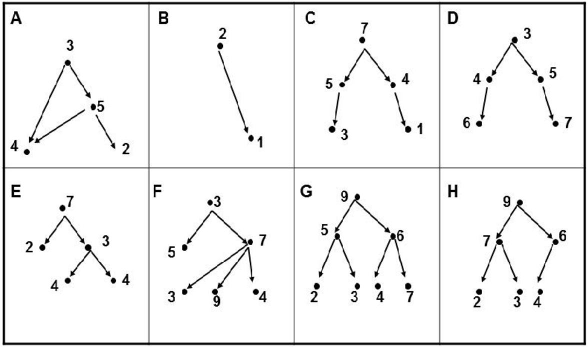
A. `(b > 9 and a < 6)`
B. `not (a < 6 or b > 8)`
C. `(a >= 6 and b <= 8)`
D. `not (a < 6 or b <= 8)`
E. `(a < 6 and b <= 8)`

Go to Exam Soln 4

M269 2014J Exam

Q 5

- **Question 5** Consider the diagrams in A–H. Nodes are represented by black dots, edges by arrows and numbers are the keys. **(4 marks)**



- Q 5 continued on next slide

Go to Exam Soln 5

M269 2014J Exam

Q 5 (contd)

- Answer the following questions. Write your answer on the line that follows each question. In each case there is at least one diagram in the answer but there may be more than one. Explanations are not required.

(a) Which of A, B, C and D **do not** show trees ?

(b) Which of E, F, G and H are binary trees ?

(c) Which of C, D, G and H are complete binary trees ?

(d) Which of C, D, G and H are binary heaps ?

Go to Exam Soln 5

M269 2014J Exam
Q 6

- **Question 6** Consider the following function, which takes an integer `n` as an argument. You can assume that `n` is positive. **(4 marks)**

```python
def twiddle(n):
    a = 5
    for i in range(n):
        for j in range(n):
            x = i * i
            y = j * j
        for k in range(n):
            z=i*k
    ans = (x + y + z) * a
    return ans
```

- Q 6 continued on next slide

Go to Exam Soln 6

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Q 6

- From the five options below, select the **one** that represents the correct combination of $T(n)$ and Big-O complexity for this function. You may assume that a step (i.e. the basic unit of computation) is the assignment statement.

A. $T(n) = n^2 + 3n + 2$ and $O(n^2)$

B. $T(n) = 2n^3 + n^2 + 2$ and $O(n^3)$

C. $T(n) = 2n^2 + n + 2$ and $O(2n^2)$

D. $T(n) = 3n^2 + 2$ and $O(n^2)$

E. $T(n) = 2n + 5$ and $O(n)$

- Now explain how you obtained $T(n)$ and the Big-O complexity.

▸ Go to Exam Soln 6

M269 2014J Exam
Q 7

(a) Given an alphabet of ACGT and the target string CAGAGAG, select the option below that represents the shift table that would be used by the Sunday string search algorithm.

A.

A	C	G	T
1	6	0	8

B.

A	C	G	T
2	1	3	0

C.

A	C	G	T
1	0	2	7

D.

A	C	G	T
2	7	1	8

▶ Q 7 continued on next slide

Go to Exam Soln 7

M269 2014J Exam
Q 7 (contd)

(b) Assuming a hash table with 20 slots, using the folding method, with 2 as the size of each part, what would be the hash value of the item 1459862913 ?

- In the box below give the hash value and indicate how you calculated your answer, showing all steps.

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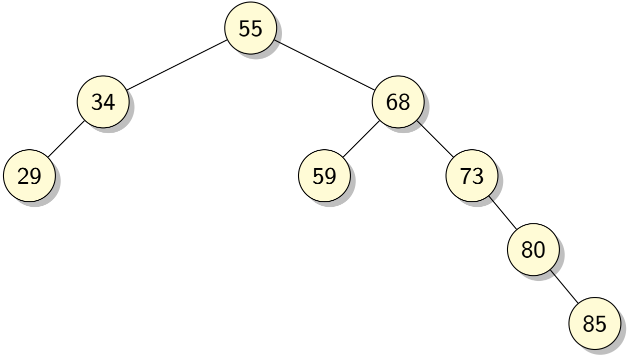
M269 2014J Exam
Q 8

- Consider the following Binary Search Tree. **(4 marks)**



- Q 8 continued on next slide

Go to Exam Soln 8

M269 2014J Exam

Q 8 (contd)

(a) Calculate the balance factors of each node in the above tree and modify the diagram to show these balance factors.

(b) Given your calculated balance factors, would this tree need to be rebalanced to be a valid AVL tree? Give your answer and a brief explanation for your answer in the box provided below.

Go to Exam Soln 8

M269 2014J Exam

Q 9

- **Question 9** Recall that the structured English for Dijkstra's algorithm is: **(5 marks)**

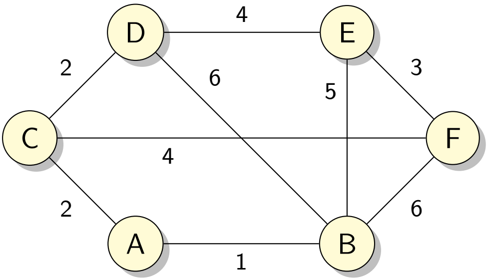
```
create priority~queue
set dist to 0 for v and dist to infinity
  for all other vertices
add all vertices to priority~queue
ITERATE while priority~queue is not empty
  remove u from the front of the queue
  ITERATE over w in the neighbours of u
    set new~distance to
      dist u + length of edge from u to w
    IF new~distance is less than dist w
      set dist w to new~distance
      change priority(w, new~distance)
```

- Q 9 continued on next slide

Go to Exam Soln 9

M269 2014J Exam
Q 9 (contd)

- Now consider the following weighted graph:



- Q 9 continued on next slide

Go to Exam Soln 9

M269 2014J Exam
Q 9 (contd)

- **Starting from vertex A**, the following table represents the distances from each vertex to A after the second line of structured English is executed for the graph given above (using the convention that the character ∞ represents infinity):

Vertex	A	B	C	D	E	F
Distance	0	∞	∞	∞	∞	∞

- *Note that neither the table above nor the subsequent tables represent the priority queue.*

- Q 9 continued on next slide

Go to Exam Soln 9

M269 2014J Exam

Q 9 (contd)

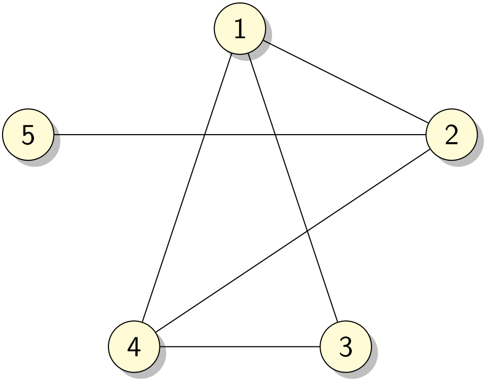
- Now, complete the appropriate boxes in the next table to show the distances after the first and second iterations of the `while` loop of the algorithm.

Vertex	A	B	C	D	E	F	
Distance	0						First iteration
Distance	0						Second iteration

Go to Exam Soln 9

M269 2014J Exam
Q 10

- **Question 10** Consider the following graph: **(4 marks)**



- Q 10 continued on next slide

Go to Exam Soln 10

M269 2014J Exam
Q 10 (contd)

- From the options below, select the **two** which show possible orders in which the vertices of the above graph could be visited in a *Breadth First Search* (BFS) starting at vertex 2:

A.

Vertex	2	3	4	5	1

B.

Vertex	2	1	4	5	3

C.

Vertex	2	1	3	4	5

D.

Vertex	2	5	1	4	3

E.

Vertex	2	5	4	3	1

F.

Vertex	2	1	3	4	5

Go to Exam Soln 10

- **Question 11** **(4 marks)**

(a) What does it mean to say that a well-formed formula (WFF) is a *contradiction* ? Use the space below for your answer.

(b) Is the following WFF a contradiction ?

$(P \wedge (Q \vee \neg Q))$

- Explain how you arrived at your answer in the space below:

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M269 2014J Exam
Q 12

- **Question 12** A particular interpretation of predicate logic allows facts to be expressed about cities and people, in particular, facts about who visited and/or liked which cities. In this interpretation, we will make use of the following two sets: **(6 marks)**

  cities = {Adelaide, San Francisco, Mumbai}

  persons = {Lin, Derren, Gabi}
- Some of the assignments in the interpretation are given below (where the symbol $\mathcal{I}$ is used to show assignment). The interpretation assigns Lin, Derren and Gabi to the constants *lin*, *derren* and *gabi*.
- $\mathcal{I}(lin) = \text{Lin} \quad \mathcal{I}(derren) = \text{Derren} \quad \mathcal{I}(gabi) = \text{Gabi}$

- Q 12 continued on next slide

Go to Exam Soln 12

M269 2014J Exam
Q 12 (contd)

- The predicates *has_visited* and likes are assigned to binary relations. The comprehensions of the relations are:

  $\mathcal{I}(\textit{has\_visited}) = \{(A, B) : \text{the person } A \text{ has visited the city } B\}$

  $\mathcal{I}(\textit{likes}) = \{(A, B) : \text{the person } A \text{ likes the city } B\}$

- The enumerations of the relations are:

  $\mathcal{I}(\textit{has\_visited}) = \{$(Lin, Adelaide), (Derren, San Francisco), (Gabi, Mumbai), (Gabi, San Francisco)$\}$

  $\mathcal{I}(\textit{likes}) = \{$(Lin, Mumbai), (Lin, San Francisco), (Derren, Mumbai), (Derren, San Francisco), (Gabi, Adelaide)$\}$

- Q 12 continued on next slide

▸ Go to Exam Soln 12

M269 2014J Exam
Q 12 (contd)

- You will find parts (a) and (b) of this question on the next page, whilst the interpretation is reproduced on the other side of this page.
- In both parts, you are given a sentence of predicate logic and asked to provide an English translation of the sentence in the box immediately following it.
- You also need to state whether the sentence is TRUE or FALSE in the interpretation that is provided on this page, and give an explanation of your answer.
- Q 12 continued on next slide

Go to Exam Soln 12

M269 2014J Exam
Q 12 (contd)

- In your explanation you need to consider any relevant values for the variables, and *show, using the interpretation above, whether they make the quantified expression TRUE or FALSE*.
- When your explanation refers to the interpretation, make sure that you use formal notation.
- So instead of saying that *Lin likes Mumbai according to the interpretation*, write:
  (Lin, Mumbai) $\in \mathcal{I}(likes)$.
- Similarly, instead of *Lin doesn't like Mumbai* you would need to write:
  (Lin, Mumbai) $\notin \mathcal{I}(likes)$.

- Q 12 continued on next slide

Go to Exam Soln 12

M269 2014J Exam
Q 12 (contd)

- **Interpretation to be used for answering Question 12**
- cities = {Adelaide, San Francisco, Mumbai}
- persons = {Lin, Derren, Gabi}
- $\mathcal{I}(\mathit{lin}) = \text{Lin} \quad \mathcal{I}(\mathit{derren}) = \text{Derren} \quad \mathcal{I}(\mathit{gabi}) = \text{Gabi}$
- $\mathcal{I}(\mathit{has\_visited})$ = {(Lin, Adelaide), (Derren, San Francisco), (Gabi, Mumbai), (Gabi, San Francisco)}
- $\mathcal{I}(\mathit{likes})$ = {(Lin, Mumbai), (Lin, San Francisco), (Derren, Mumbai), (Derren, San Francisco), (Gabi, Adelaide)}
- Q 12 continued on next slide

Go to Exam Soln 12

M269 2014J Exam

Q 12 (contd)

(a) $\exists X.\neg(likes(lin, X) \wedge likes(derren, X))$

can be translated into English as:

- This sentence is ____ (choose from TRUE/FALSE), because:

(b) $\forall X.(likes(gabi, X) \vee has\_visited(gabi, X))$

can be translated into English as:

- This sentence is ____ (choose from TRUE/FALSE), because:

Go to Exam Soln 12

M269 2014J Exam
Q 13

- **Question 13** A database contains the following tables, *production_line* and *product*. **(6 marks)**

production_line	
id	*unit*
Line1	Sportscar
Line2	SUV
Line3	Bus
Line4	Tractor
Line5	Aeroplane

product	
type	*unit_price*
SUV	50000
Sportscar	200000
Bus	250000
Tractor	50000
Aeroplane	30000000

- Q 13 continued on next slide

Go to Exam Soln 13

(a) For the following SQL query, give the table returned by the query.

```
SELECT id, unit_price
FROM production_line CROSS JOIN product
WHERE unit = type AND unit_price < 300000;
```

(b) Write an SQL query which answers the question *Which products cost exactly 50000?* The answer should be the following table:

type
SUV
Tractor

Go to Exam Soln 13

M269 2014J Exam
Q 14

- **Question 14** Consider the following axiom schema and rules: **(4 marks)**

Axiom schema	$\{\boldsymbol{A}\} \vdash \boldsymbol{A}$
Rules	$\dfrac{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \wedge \boldsymbol{B}}{\boldsymbol{\Gamma} \vdash \boldsymbol{A}}$ ($\wedge$-elimination left)
	$\dfrac{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \wedge \boldsymbol{B}}{\boldsymbol{\Gamma} \vdash \boldsymbol{B}}$ ($\wedge$-elimination right)
	$\dfrac{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \quad \boldsymbol{\Gamma} \vdash \boldsymbol{B}}{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \wedge \boldsymbol{B}}$ ($\wedge$-introduction)
	$\dfrac{\boldsymbol{\Gamma} \cup \{\boldsymbol{A}\} \vdash \boldsymbol{B}}{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \rightarrow \boldsymbol{B}}$ ($\rightarrow$-introduction)
	$\dfrac{\boldsymbol{\Gamma} \vdash \boldsymbol{A} \quad \boldsymbol{\Gamma} \vdash \boldsymbol{A} \rightarrow \boldsymbol{B}}{\boldsymbol{\Gamma} \vdash \boldsymbol{B}}$ ($\rightarrow$-elimination)

- Q 14 continued on next slide

Go to Exam Soln 14

M269 2014J Exam
Q 14

- Complete the following proof by filling in the two boxes. You can use any of the above as appropriate.

1.	$\{(V \land W)\} \vdash (V \land W)$	[Axiom schema]
2.	$\boxed{?? \qquad\qquad ??}$	[1 $\land$-elimination right]
3.	$\varnothing \vdash (V \land W) \to W$	$\boxed{?? \qquad\qquad ??}$

Go to Exam Soln 14

M269 2014J Exam
Q 15

(a) Complete the following sentence in the box below:

The statement *If a computational problem is in NP, then it must be intractable* may be false because

- Q 15 continued on next slide

Go to Exam Soln 15

M269 2014J Exam
Q 15 (contd)

(b) Consider the following Turing Machine:

	\$	EMPTY
0		WRITE \$ MOVE RIGHT NEXT STATE 1
1	WRITE blank MOVE LEFT NEXT STATE 0	WRITE \$ MOVE RIGHT NEXT STATE 0

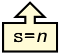
- Q 15 continued on next slide

Go to Exam Soln 15

M269 2014J Exam

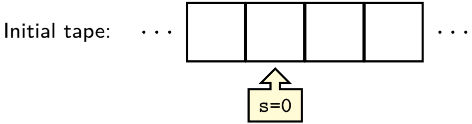
Q 15 (contd)

- Assume that the starting state is 0 and that the input tape consists of empty squares.
- One square is marked as the current square with the tape head, shown here as



where n is the current state

- So, initially the tape looks as follows:



- Note it the original exam the tape cells and head are denoted by ASCII symbols
- Q 15 continued on next slide

Go to Exam Soln 15

M269 2014J Exam

Q 15 (contd)

- Using the same notation, write down what the tape looks like after each of the next two steps of the computation.
- Use the boxes below for this.
- After the first step

First step:	$\cdots$					$\cdots$

- After the second step

Second step:	$\cdots$					$\cdots$

▸ Go to Exam Soln 15

M269 Exam 2014J
Q Part2

- Answer every question in this part.
- The marks for each question are given at the end of the question.
- Answers to this part should be written in the separate answer book.

▸ Go to Exam 2014J Soln Part2

M269 2014J Exam
Q 16

- The Book Brigade is a start-up online bookseller specialising in electronic books.
- The company asks customers to rate the books they have read on a scale of 1 (dross) to 10 (magnificent), and maintains data in two sequences, B and R.

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam
Q 16 (contd)

1. B is an unsorted sequence of ISBNs (International Standard Book Numbers — a unique numerical code for every book published), together with the title of the book the ISBN denotes. Thus each item in B is itself a 2-tuple with items: (1) the ISBN, and (2) the title.
2. R is an unsorted sequence of ISBNs with, for each item, a list of customer ratings for that book. Each item in R is also a 2-tuple with items: (1) the ISBN, and (2) a sequence of ratings.

- Some books in B may not have been rated, and these will not appear in R. Moreover, the order of books in R is not necessarily the same as the order in B.

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam

Q 16 (contd)

(a) The company requires a computer system that generates a list of book titles that have been rated, with the average rating of each. Unrated books should not appear in this list. **(5 marks)**

(i) Using the following template, formally state this as a computational problem, in the style adopted by M269.

Name: BookRatings
Inputs:
Outputs:

(ii) Suggest one possible postcondition for this computational problem.

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam
Q 16 (contd)

(b) Sketch out an initial insight for a computational solution of the BookRatings problem. **(6 marks)**

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam

Q 16 (contd)

(c) Many potential solutions to the BookRatings problem may be inefficient, arising from the fact that neither B nor R are sorted.

- The Book Brigade now require a more efficient solution which will require that both these lists, and the list of results returned by the system, will be sorted.
- Since all these lists are likely to be very long, an efficient sorting algorithm is required, and it has been decided to use *Quicksort*.

(i) Write a short paragraph explaining the process by which Quicksort sorts a list in-place; **(5 marks)**

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam

Q 16 (contd)

(c) **(ii)** The list

44	55	12	42	94	18	6	67

- is being sorted using an in-place Quicksort, with the first item of a partition being chosen as the pivot.
- Draw diagrams illustrating the pivot and the left and right pointers: **(5 marks)**

(1) at the very start (of the first pass),

(2) immediately before the first swap and

(3) immediately before the second swap.

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam
Q 16 (contd)

(d) Storing data items (e.g. book title, ratings) that are associated with a key (e.g. ISBN) in a list, as Book Brigade have opted to do, can be problematic, in particular because retrieval may be slow when there are very large numbers of keys. A more suitable means of storage is to associate keys with their data in a *hash table*.

- Q 16 continued on next slide

Go to Soln 16

M269 2014J Exam
Q 16 (contd)

(d) (contd) Write two short paragraphs such that

- the first paragraph explains the basic principles of hash tables and hashing (about 5 sentences), and
- the second paragraph outlines one problem that might arise from hashing, and mentions a strategy for addressing the aforementioned problem (about 3 sentences).

- You do not need to describe the details of specific hashing functions or strategies. **(4 marks)**

Go to Soln 16

M269 2014J Exam
Q 17

- An educational TV show is planning to do a presentation on the Halting Problem. To brief them, you've been asked to write a report for the producers. Assume that the producers do not have a background in computer science.
- Q 17 continued on next slide

Go to Soln 17

M269 2014J Exam

Q 17 (contd)

- Your report **must** have the following structure:

1. A suitable title
2. A paragraph setting the scene and explaining in layperson's terms what is meant by *the Halting Problem* [about two sentences]
3. One paragraph in which you describe the relationship between the Halting Problem and a Turing Machine [about two sentences]
4. One paragraph in which you describe (and give an example in layperson's terms of) proof by contradiction [about three sentences]
5. A conclusion, giving reasons, about the significance of the Halting Problem not being computable [one sentence]

- Q 17 continued on next slide

Go to Soln 17

M269 2014J Exam
Q 17 (contd)

- Note that a significant number of marks will be awarded for coherence and clarity, so avoid abrupt changes of topic and make sure your sentences fit together to tell an overall story.
- Allow up to four additional sentences to ensure this, although note that the numbers of sentences specified, in points 1 to 5 above and in this paragraph, are for guidance only. **(15 marks)**

Go to Soln 17

M269 2014J Exam

Solns

- The solutions given below are not official solutions
- For some questions, alternatives are given — a student would only have to provide one

Go to Exam Q s

M269 2014J Exam
Soln Part1

- Part 1 solutions

Go to Exam Q Part1

M269 2014J Exam
Soln 1

- A, C

Go to Exam Q 1

M269 2014J Exam
Soln 2

A 5 model

B 1 represented by

C 2 ignores detail of

D 6 part of reality

- See Unit 1 section 3.2 *Computational thinking and abstraction*

Go to Exam Q 2

M269 2014J Exam
Soln 3

- The sorted part of the list is filled in pale blue with the next item to be inserted in pale red

(Pass 1)	26	54	93	17	77	44	55	31
(Pass 2)	26	54	93	17	77	44	55	31
(Pass 3)	17	26	54	93	77	44	55	31
(Pass 4)	17	26	54	77	93	44	55	31

Go to Exam Q 3

M269 2014J Exam

Soln 4

(a) Truth table

P	Q	$\neg Q$	$P \vee \neg Q$	$\neg(P \vee \neg Q)$	$Q \wedge \neg(P \vee \neg Q)$
T	T	F	T	F	F
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	T	F	F

(b) The only row that has True in the final column for the guard is

$P = F$ and $Q = T$

- This is equivalent to

  $\neg P \wedge Q$

- $\rightarrow \neg(P \vee \neg Q)$ hence answer D

Go to Exam Q 4

M269 2014J Exam

Soln 5

(a) In a tree, there is a unique path from the root to each node (graph theory version)

- A is not a tree since 4 can be reached 3–4 or 3–5–4
- B, C, D are trees

(b) In a binary tree, each node has at most two children (graph theory version) Note also that in a binary tree each child node is either a left or a right child. The inductive definition of a binary tree: a binary tree is either an empty tree or a node with two subtrees

- E, G, H are binary trees

(c) In a complete binary tree, every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

- G, H are complete trees.

(d) Binary heaps are complete binary trees that satisfy the heap order property. This property requires that the key of a node is smaller or equal to the key of its children for min heaps or greater or equal for max heap

- H is a binary heap

Go to Exam Q 5

M269 2014J Exam
Soln 6

- D $T(n) = 3n^2 + 2$ and $O(n^2)$
- Explanation:
- 2 assignment statements outside loops
- 3 assignment statements inside two nested loops which are each executed n times ($3n^2$)

Go to Exam Q 6

M269 2014J Exam
Soln 7

(a) D

- The shift table for the Sunday Quick Search algorithm:
  - If the character does not appear in the target string T, the shift distance is one more than the length of T
  - If the character does appear in T the shift distance is the first position at which it appears, counting from right to left and starting at 1

Go to Exam Q 7

M269 2014J Exam
Soln 7 (contd)

(b) Hash value $= 1$

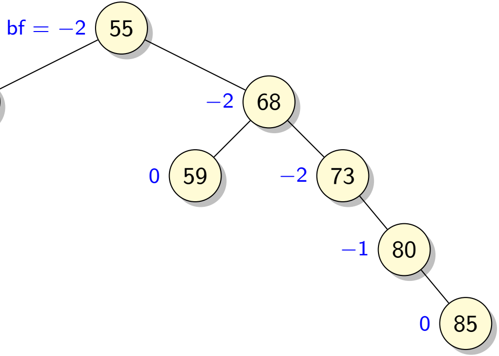
since $(14 + 59 + 86 + 29 + 13) \% 20 = 1$

Go to Exam Q 7

M269 2014J Exam

Soln 8

(a) Tree with balance factors



- Soln 8 continued on next slide

Go to Exam Q 8

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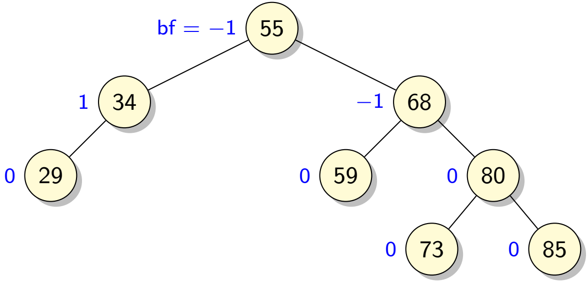
Soln 8 (contd)

(b) The tree is not a valid AVL tree since some of the balance factors are outside the range $[-1, 0, +1]$

- The tree must have become unbalanced as a result of inserting 85
- Rebalancing would mean a left rotation around the node 73 as below (this diagram was not required in the exam)



Go to Exam Q 8

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Soln 9

- The completed table

Vertex	A	B	C	D	E	F	
Distance	0	1	2	∞	∞	∞	First iteration
Distance	0	1	2	7	6	7	Second iteration

- The node being processed has a red background
- Nodes with a final label have a green background
- Neighbours of the node being processed have blue text

Go to Exam Q 9

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Soln 9 (contd)

- The complete iterations

Vertex	A	B	C	D	E	F	
Distance	0	1	2	∞	∞	∞	First iteration
Distance	0	1	2	7	6	7	Second iteration
Distance	0	1	2	4	6	6	Third iteration
Distance	0	1	2	4	6	6	Fourth iteration
Distance	0	1	2	4	6	6	Fifth iteration
Distance	0	1	2	4	6	6	Sixth iteration

Go to Exam Q 9

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Soln 10

- B, D
- A *Breadth First Search* (BFS) from 2 must visit its neighbours in some order first

Go to Exam Q 10

M269 2014J Exam
Soln 11

(a) A WFF is a *contradiction* if it is *False* in every *interpretation* — an interpretation is an assignment of meaning to the symbols of a formal language — here it is assignment of truth values to P and Q

(b) Truth table

P	Q	$\neg Q$	$Q \vee \neg Q$	$(P \wedge (Q \vee \neg Q))$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

- The statement is not a contradiction — it is satisfiable — it has the same truth value as P

Go to Exam Q 11

M269 2014J Exam

Soln 12

(a) $\exists X.\neg(likes(lin, X) \wedge likes(derren, X))$

- There is a city that both Lin and Derren like
- *True* since X could be *Mumbai* or *San Francisco*

(b) $\forall X.(likes(gabi, X) \vee has\_visited(gabi, X))$

- *Gabi* either likes or has visited all cities
- *True* since *Gabi* likes *Adelaide* and has visited the others.

Go to Exam Q 12

M269 2014J Exam

Soln 13

(a) The output table

id	*unit_price*
Line1	200000
Line2	50000
Line3	250000
Line4	50000

(b) SQL query

```
SELECT type
FROM product
WHERE unit_price = 50000;
```

Go to Exam Q 13

M269 2014J Exam
Soln 14

- Completed proof

1.	$\{(V \wedge W)\} \vdash (V \wedge W)$	[Axiom schema]
2.	$\boxed{\{(V \wedge W)\} \vdash W}$	[1 $\wedge$-elimination right]
3.	$\varnothing \vdash (V \wedge W) \to W$	$\boxed{2 \to\text{-introduction}}$

- Note $\varnothing$ is a symbol for the empty set (in LaTeX `\varnothing`)
- You could also use $\{\}$ (or even leave a blank, but that would not be good practice)

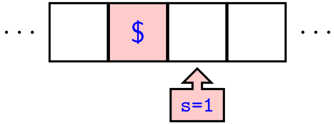
Go to Exam Q 14

M269 2014J Exam
Soln 15

(a) P is a subset of NP but we do not know if it is a proper subset — so the problem may be in P

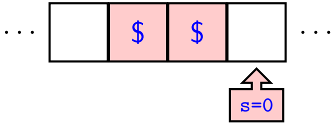
(b) After the first step

First step:



- After the second step

Second step:



Go to Exam Q 15

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Soln Part2

- Part 2 solutions

Go to Exam 2014J Q Part2

M269 2014J Exam

Soln 16

(a) **(i)**

Name: BookRatings

Inputs: An unsorted sequence of tuples $B = (b_1, b_2, \ldots, b_n)$ where $b_n = (i_n, t_n)$ and ISBN, i_n, and title, t_n, are strings.

An unsorted sequence of tuples $R = (r_1, r_2, \ldots, r_k)$ where $r_k = (i_k, l_k)$ and i_k is an ISBN and l_k is a list of ratings (0–10)

Outputs: a list of tuples $O = (o_1, o_2, \ldots, o_p)$ where $o_p = (t_p, a_p)$ and t_p is a title and a_p is a real number between 0 and 10.

- **(ii)** The length of R must equal the length of O, $k = p$

- Soln 16 continued on next slide

Go to Q 16

M269 2014J Exam

Soln 16 (contd)

(b) Generate the elements of O by iterating over R

- For each element of R, find the title from the ISBN by iterating over B
- and calculate the average rating from the list of ratings

- Soln 16 continued on next slide

Go to Q 16

M269 2014J Exam
Soln 16 (contd)

(c) **(i)** *Quicksort* chooses an item in the list to be the *pivot* item.

- The algorithm partitions the list into two sublists
- One list comprises items in the list less than the pivot
- The other list comprises the elements in the list greater than or equal to the pivot
- Recursively sort the sub lists (with Quicksort)
- Join the sorted sub lists together with the pivot
- An array based implementation uses two pointers, *leftMK* and *rightMK*, to do the partitioning in place

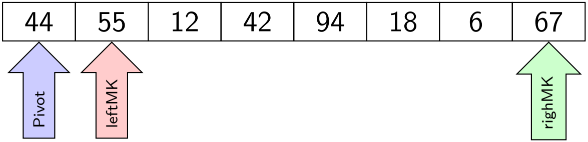
- Soln 16 continued on next slide

Go to Q 16

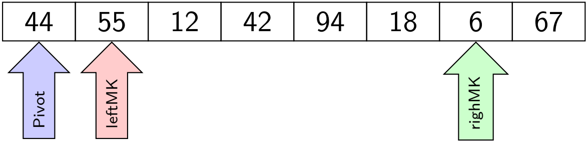
M269 2014J Exam

Soln 16 (contd)

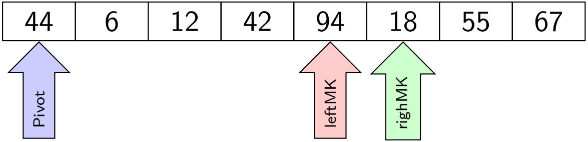
- At the start of the first pass



- Before the first swap



- Before the second swap



- Soln 16 continued on next slide

M269 2014J Exam

Soln 16 (contd)

(d) Hash function and hash tables

- Hash function maps each input key to a hash value (or slot)
- Perfect hash function maps each key to a different hash value
- Limited storage leads to hash functions having collisions — a hash function mapping two keys to the same slot
- Hash function collisions result in the need to either store multiple items in a single slot (closed table) or open addressing/open tables that use some mechanism to find a free slot

Go to Q 16

M269 2014J Exam

Soln 17

- **Title** The Significance of the Halting Problem
- **Definition** Can we write an algorithm (a program) that will check whether any other program (algorithm) will terminate (halt) or loop forever
- We need a formal definition of *algorithm* — hence the need for the definition of Turing machine
- Turing machine (and the equivalent Lambda Calculus and others) formalises our idea of functions that are computable — functions that can be calculated.
- Proof by argument similar to Cantor's Diagonal argument and Proof by contradiction
- Direct consequences in mathematics and computer science — the Entscheidungsproblem from David Hilbert 1928 — can we have an algorithm that takes a statement in first order logic and checks if it is valid

Go to Q 17