M269 Exams

Prsntn 2013J Exam

Phil Molyneux

5 September 2016

M269 2013J Exam
Qs

- M269 Algorithms, Data Structures and Computability
- Presentation 2013J Exam
- Date Wednesday, 4 June 2015 Time 14:30–17:30
- There are **TWO** parts to this examination. You should attempt all questions in **both** parts
- **Part 1** carries **60 marks — 100 minutes**
- **Part 2** carries **40 marks — 70 minutes**
- **Note** see the original exam paper for exact wording and formatting — these slides and notes may change some wording and formatting

Go to Exam Solns

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Q Part1

- Answer every question in this part.
- Answers to questions in this part should be written on this paper in the spaces provided, or in the case of multiple-choice questions you should tick the appropriate box(es).
- If you tick more boxes than indicated for a multiple choice question, you will receive **no** marks for your answer to that question.

Go to Exam Soln Part1

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Q 1

- Which **two** of the following statements are **true**?

A. A decision problem is any problem stated in a formal language.

B. A computational problem is a problem that is expressed sufficiently precisely that it is possible to build an algorithm that will solve all instances of that problem.

C. An algorithm consists of a precisely stated, step-by-step list of instructions.

D. Computational thinking is the skill to formulate a problem as a computational problem, and then construct a good computational solution, in the form of an algorithm, to solve this problem, or explain why there is no such solution. **(2 marks)**

Go to Exam Soln 1

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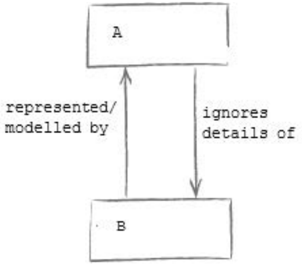
Q 2

- **Question 2** The general idea of abstraction as modelling can be shown with the following diagram.

(2 marks)



- Q 2 continued on next slide

Go to Exam Soln 2

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Q 2 (contd)

- Complete the diagram above by adding an appropriate label (one of the numbers 1 to 4) in the space indicated by **A** and one in the space indicated by **B**. The possible answers are shown as 1 to 4 below.
  1. A car crash test dummy in the real world
  2. An action man doll in the real world
  3. A real car in the real world (after crashing)
  4. A real driver in the real world
- *The exam question had some pictures next to the texts*

Go to Exam Soln 2

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Q 3

- **Question 3** A binary search is being carried out on the list shown below for item 67: **(4 marks)**

  `[12,16,17,24,41,49,51,62,67,69,75,80,89,97,101]`

- For each pass of the algorithm, draw a box around the items in the partition to be searched during that pass, continuing for as many passes as you think are needed.
- We have done the first pass for you showing that the search starts with the whole list. Draw your boxes below for each pass needed; you may not need to use all the lines below. *(The question had 8 rows)*

(Pass 1) [$\boxed{\texttt{12,16,17,24,41,49,51,62,67,69,75,80,89,97,101}}$]
(Pass 2) `[12,16,17,24,41,49,51,62,67,69,75,80,89,97,101]`
(Pass 3) `[12,16,17,24,41,49,51,62,67,69,75,80,89,97,101]`

Go to Exam Soln 3

M269 2013J Exam
Q 4

- **Question 4** Consider the guard in the following Python while loop header: **(4 marks)**

```
while (a < 6 and b > 8) or not(a >= 6 or b <= 8):
```

(a) Make the following substitutions:

P represents `a < 6`
Q represents `b > 8`

Then complete the following truth table:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$	$(P \wedge Q) \vee \neg(\neg P \vee \neg Q)$
F	F						
F	T						
T	F						
T	T						

- Q 4 continued on next slide

Go to Exam Soln 4

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Q 4

(b) Use the results from your truth table to choose which one of the following expressions could be used as the simplest equivalent to the above guard.

A. `(a < 6 and b > 8)`

B. `not(a < 6 and b > 8)`

C. `(a >= 6 or b <= 8)`

D. `(a >= 6 and b <= 8)`

E. `(a < 6 and b <= 8)`

Go to Exam Soln 4

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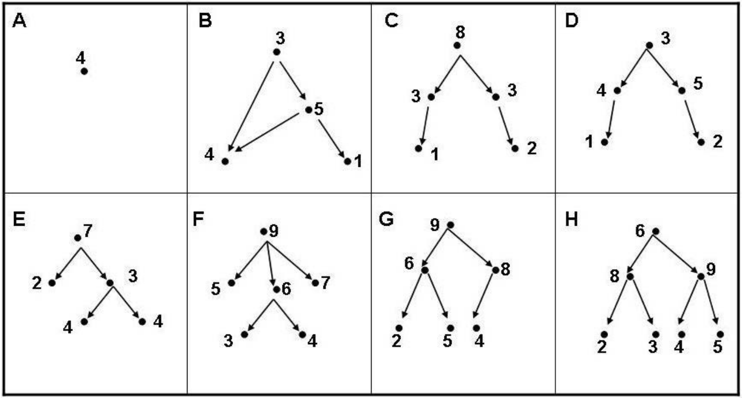
Q 5

- **Question 5** Consider the following diagrams A–H. Nodes are represented by black dots and edges by arrows. The numbers represent a node's key. **(4 marks)**



- Q 5 continued on next slide

Go to Exam Soln 5

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Q 5 (contd)

- Answer the following questions. Write your answer on the line that follows each question. In each case there is at least one diagram in the answer but there may be more than one. Explanations are not required.

(a) Which of A, B, C and D **do not** show trees ?

(b) Which of E, F, G and H are binary trees ?

(c) Which of C, D, G and H are complete binary trees ?

(d) Which of C, D, G and H are binary heaps ?

Go to Exam Soln 5

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Q 6

- Consider the following function, which takes a list as an argument. You may assume that the list contains a number of integer values and is not empty.

```
def average(aList):
    n = len(aList)
    total = 0
    for item in aList:
        total = total + item
    mean = total / n
return mean
```

- Q 6 continued on next slide

Go to Exam Soln 6

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Q 6 (contd)

- From the five options below, select the **one** that represents the correct combination of $T(n)$ and Big-O complexity for this function. You may assume that a step (i.e. the basic unit of computation) is the assignment statement.

A. $T(n) = 3 + n^2$ and $O(n^2)$

B. $T(n) = n + 2$ and $O(n^2)$

C. $T(n) = 2n + 2$ and $O(n)$

D. $T(n) = 3n + n^2$ and $O(n^2)$

E. $T(n) = n + 3$ and $O(n)$

Go to Exam Soln 6

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Q 7

- In the KMP algorithm, for each character in the target string T we identify the longest substring of T ending with that character which matches a prefix of the target string.
- These lengths are stored in what is known as a **prefix table** (which in Unit 4 we represented as a list).
- Consider the target string T

A	B	A	C	A	B	A	C	A	C

- Below is an incomplete prefix table for the target string given above. Complete the prefix table by writing the missing numbers in the appropriate boxes.

0		1	0		2		4		0

Go to Exam Soln 7

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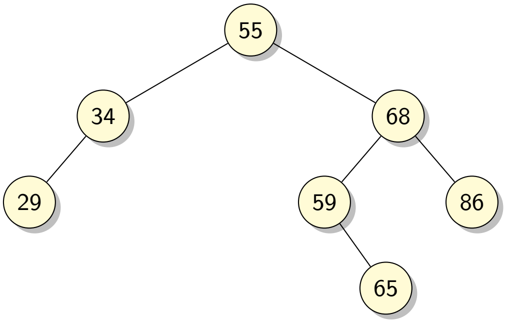
Q 8

- **Question 8** **(4 marks)**

(a) Consider the following Binary Search Tree.



- Modify (draw on) the above Binary Search Tree to insert a node with a key of 57.
- Q 8 continued on next slide

Go to Exam Soln 8

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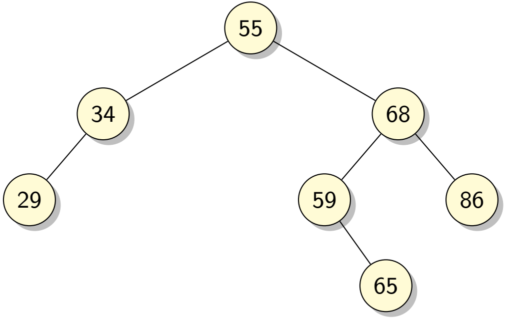
Q 8 (contd)

(b) Once again, consider the same Binary Search Tree.



- Calculate the balance factors of each node in the tree above and modify the diagram to show these balance factors.

Go to Exam Soln 8

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Q 9

- Recall that the structured English for Dijkstra's algorithm is:

```
create priority~queue
set dist to 0 for v and dist to infinity
  for all other vertices
add all vertices to priority~queue
ITERATE while priority~queue is not empty
  remove u from the front of the queue
  ITERATE over w in the neighbours of u
    set new~distance to
      dist u + length of edge from u to w
    IF new~distance is less than dist w
      set dist w to new~distance
      change priority(w, new~distance)
```

- Q 9 continued on next slide

Go to Exam Soln 9

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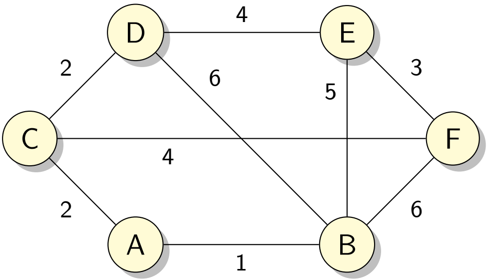
Q 9 (contd)

- Now consider the following weighted graph:



- Q 9 continued on next slide

Go to Exam Soln 9

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Q 9 (contd)

- **Starting from vertex B**, the following table represents the distances from each vertex to A after the second line of structured English is executed for the graph given above (using the convention that a blank cell represents infinity):

Vertex	A	B	C	D	E	F
Distance		0				

- *Note that neither the table above nor the subsequent tables represent the priority queue.*

- Q 9 continued on next slide

Go to Exam Soln 9

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Q 9 (contd)

- Now, complete the appropriate boxes in the next table to show the distances after the first and second iterations of the `while` loop of the algorithm.

Vertex	A	B	C	D	E	F	
Distance		0					First iteration
Distance		0					Second iteration

Go to Exam Soln 9

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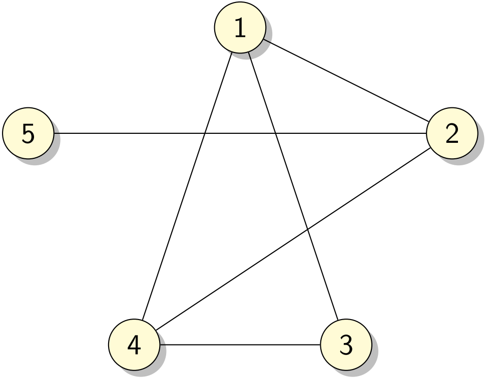
Q 10

- **Question 10** Consider the following graph: **(4 marks)**



- Q 10 continued on next slide

Go to Exam Soln 10

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Q 10

- Complete the table below to show the order in which the vertices of the above graph could be visited in a Depth First Search (DFS) starting at vertex 3 and always choosing first the leftmost not yet visited vertex (as seen from the current vertex):

Vertex	3				

▸ Go to Exam Soln 10

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Q 11

- **Question 11** **(4 marks)**

(a) What does it mean to say that a well-formed formula (WFF) is *satisfiable* ? Use the space below for your answer

(b) Is the following WFF satisfiable ?

$$(P \rightarrow (Q \rightarrow P)) \vee \neg R$$

- Explain how you arrived at your answer

Go to Exam Soln 11

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Q 12

- A particular interpretation of predicate logic allows facts to be expressed about films that people have seen, and of which they own copies.
- Some of the assignments in the interpretation are given below (where the symbol $\mathcal{I}$ is used to show assignment).
- The interpretation assigns *Jane*, *John* and *Saira* to the constants *jane*, *john* and *saira*.

$$\begin{aligned}\mathcal{I}(\text{jane}) &= \text{Jane}\\ \mathcal{I}(\text{john}) &= \text{John}\\ \mathcal{I}(\text{saira}) &= \text{Saira}\end{aligned}$$

- Q 12 continued on next slide

Go to Exam Soln 12

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Q 12 (contd)

- The predicates *owns* and *has_seen* are assigned to binary relations. The comprehensions of the relations are:
  - $\mathcal{I}$(owns) = {(A,B): the person A owns a copy of film B}
  - $\mathcal{I}$(has_seen) = {(A,B): the person A has seen film B}
- The enumerations of the relations are:

$$\begin{aligned}
\mathcal{I}(\text{owns}) &= \{(\text{Jane, Django}), (\text{Jane, Casablanca}), \\
&\quad (\text{John, Jaws}), (\text{John, The Omen}), (\text{John, El Topo}), (\text{Saira, El Topo}), (\text{Saira, Casablanca})\} \\
\mathcal{I}(\text{has_seen}) &= \{(\text{Jane, Django}), (\text{Jane, Candide}), (\text{Jane, Casablanca}), \\
&\quad (\text{John, The Omen}), (\text{John, El Topo}), (\text{Saira, Django}), (\text{Saira, The Omen})\}
\end{aligned}$$

- Q 12 continued on next slide

Go to Exam Soln 12

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Q 12 (contd)

- Parts (a) and (b) of this question are on the next page.
- In both parts, you are given a sentence of predicate logic and asked to provide an English translation of the sentence in the box immediately following it.
- You also need to state whether the sentence is TRUE or FALSE in the interpretation that is provided on this page, and give an explanation of your answer.
- In your explanation you need to consider any relevant values for the variable X, *and show, using the interpretation above, whether it makes the quantified expression TRUE*.

- Q 12 continued on next slide

▸ Go to Exam Soln 12

M269 2013J Exam
Q 12 (contd)

(a) $\forall X.(owns(saira, X) \rightarrow has\_seen(saira, X))$

can be translated in English as:

- This sentence is TRUE/FALSE because:

(b) $\exists X.(has\_seen(jane, X) \wedge owns(jane, X))$

can be translated in English as:

- This sentence is TRUE/FALSE because:

Go to Exam Soln 12

M269 2013J Exam
Q 13

- **Question 13** A database contains the following tables, *oilfield* and *operator* **(6 marks)**

oilfield	
name	*production*
Warga	3
Lolli	5
Tolstoi	0.5
Dakhun	2
Sugar	3

operator	
company	*field*
Amarco	Warga
Bratape	Lolli
Rosbif	Tolstoi
Taqar	Dakhun
Bratape	Sugar

- Q 13 continued on next slide

▸ Go to Exam Soln 13

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Q 13

- For each of the following SQL queries, give the table returned by the query

(a)

```
SELECT *
FROM operator;
```

(b)

```
SELECT name, production
FROM oilfield
WHERE production > 2;
```

(c)

```
SELECT name, production, company
FROM oilfield CROSS JOIN operator
WHERE name = field;
```

Go to Exam Soln 13

M269 2013J Exam
Q 14

- **Question 14** Consider the following axiom schema and rules: **(4 marks)**

Axiom schema	$\{A\} \vdash A$	
Rules	$\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}$	($\wedge$-elimination left)
	$\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}$	($\wedge$-elimination right)
	$\dfrac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}$	($\wedge$-introduction)
	$\dfrac{\Gamma \cup \{A\} \vdash B}{\Gamma \vdash A \rightarrow B}$	($\rightarrow$-introduction)
	$\dfrac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B}$	($\rightarrow$-elimination)

- Q 14 continued on next slide

Go to Exam Soln 14

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Q 14 (contd)

- Complete the following proof by filling in the two boxes. You can use any of the above as appropriate.

1.	$\{P \wedge (Q \wedge R)\} \vdash P \wedge (Q \wedge R)$	[Axiom schema]
2.	$\boxed{??\qquad\qquad ??}$	[1,$\wedge$-elimination left]
3.	$\varnothing \vdash (P \wedge (Q \wedge R)) \rightarrow P$	$\boxed{??\qquad\qquad ??}$

Go to Exam Soln 14

M269 2013J Exam
Q 15

- **Question 15** Which *two* of the following statements are true? **(2 marks)**

A. A Turing Machine is a mathematical model of computational problems.

B. If the lower bound for a computational problem is $O(n^2)$, then there is an algorithm that solves the problem and which has complexity $O(n^2)$.

C. Searching a sorted list is not in the class NP.

D. The decision Travelling Salesperson Problem is NP-complete.

E. There is no known tractable quantum algorithm for solving a known NP-complete problem.

▸ Go to Exam Soln 15

M269 2013J Exam
Q Part2

- Answer every question in this part.
- The marks for each question are given at the end of the question.
- Answers to this part should be written in the separate answer book

Go to Exam Soln Part2

M269 2013J Exam
Q 16

- Multipart question
- Specification of program, data structures, pre and post conditions
- Write a small program
- Give the complexity of the small program
- Give insight into a sorting algorithm
- Give insight into insertion into a binary search tree
- See notes version for text

Go to Exam Soln 16

M269 2013J Exam
Q 17

- Write short report on a computational topic
- Suitable title for the topic and audience
- Paragraph setting the scene — the context of the topic
- Paragraph describing the topic
- Paragraph on the role the topic plays in some area
- Conclusions justifying the importance of the topic
- See notes version for text.

Go to Exam Soln 17

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Solns

- The solutions given below are not official solutions
- For some questions, alternatives are given — a student would only have to provide one

Go to Exam Qs

M269 2013J Exam
Soln Part1

- Part 1 solutions

Go to Exam Q Part1

M269 2013J Exam
Soln 1

- Options C and D are true.
- Option A is wrong because decision problems have to have a yes-no answer.
- Option B is wrong because there are computational problems that we can state and build algorithms for, but cannot always be solved.

Go to Exam Q 1

M269 2013J Exam

Soln 2

- A real driver is modelled by a car crash test dummy, so A = 1 and B = 4

Go to Exam Q 2

M269 2013J Exam
Soln 3

- The complete binary search:

  (Pass 1) [$\boxed{\texttt{12,16,17,24,41,49,51,62,67,69,75,80,89,97,101}}$]

  (Pass 2) [12,16,17,24,41,49,51,62,$\boxed{\texttt{67,69,75,80,89,97,101}}$]

  (Pass 3) [12,16,17,24,41,49,51,62,$\boxed{\texttt{67,69,75}}$,80,89,97,101]

  (Pass 4) [12,16,17,24,41,49,51,62,$\boxed{\texttt{67}}$,69,75,80,89,97,101]

Go to Exam Q 3

M269 2013J Exam

Soln 4

(a) The completed truth table:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$	$(P \wedge Q) \vee \neg(\neg P \vee \neg Q)$
F	F	T	T	F	T	F	F
F	T	T	F	F	T	F	F
T	F	F	T	F	T	F	F
T	T	F	F	T	F	T	T

(b) A is the simplest equivalent of the guard given (and the only equivalent).

Go to Exam Q 4

M269 2013J Exam

Soln 5

(a) B is not a tree; it has more than one route from node 3 to node 4.

(b) E, G, and H are binary trees; (no more than 2 children per node).

(c) G, and H are complete binary trees.

(d) Only G is a heap; (complete binary tree, and parent nodes $>$ children).

▸ Go to Exam Q 5

M269 2013J Exam
Soln 6

- Option E is correct.
- The function does three assignments once per call, and one assignment for each of the n items in the argument, hence $T(n) = n + 3$.

Go to Exam Q 6

M269 2013J Exam

Soln 7

- The complete prefix table, with new entries in blue:

0	0	1	0	1	2	3	4	5	0

- Here is the target, prefix and shift:

	A	B	A	C	A	B	A	C	A	C	Target string, t
	0	1	2	3	4	5	6	7	8	9	Position (Index), p
0	1	2	3	4	5	6	7	8	9	10	Match, q
	0	0	1	0	1	2	3	4	5	0	$\text{prefixTable}(t, p)$
1	1	2	2	4	4	4	4	4	4	10	$\text{shift}(t, q)$

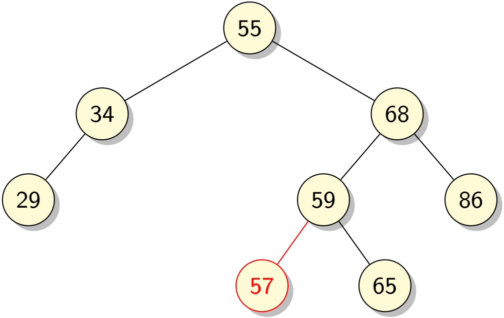
- The shift function takes the *target string*, t, and the number of characters matched, q.
- $\text{shift}(t, 0) = 1$
- $\text{shift}(t, q) = q - \text{prefixTable}(t, q - 1)$

Go to Exam Q 7

M269 2013J Exam

Soln 8

(a) Answer, with inserted node shown in red



- Soln 8 continued on next slide

Go to Exam Q 8

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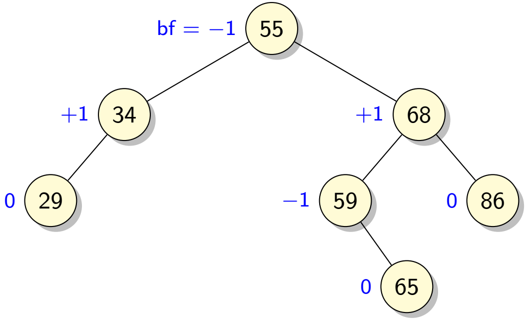
Soln 8 (contd)

(b) Answer, with balance factors shown in blue



Go to Exam Q 8

M269 2013J Exam

Soln 9

- The completed table

Vertex	A	B	C	D	E	F	
Distance	1	0		6	5	6	First iteration
Distance	1	0	3	6	5	6	Second iteration

Go to Exam Q 9

M269 2013J Exam

Soln 10

- Depth First Search (DFS) starting at vertex 3 and always choosing first the leftmost not yet visited vertex (as seen from the current vertex):

Vertex	3	4	1	2	5

- Notice the ambiguity about the term *leftmost* — an alternative view could have been:

Vertex	3	1	4	2	5

Go to Exam Q 10

M269 2013J Exam
Soln 11

(a) A WFF is *satisfiable* if it is possible to find an interpretation that makes the formula true.

(b) Truth table for the WFF

P	Q	R	$Q \rightarrow P$	$P \rightarrow (Q \rightarrow P)$	$\neg R$	$(P \rightarrow (Q \rightarrow P)) \vee \neg R$
F	F	F	T	T	T	T
F	F	T	T	T	F	T
F	T	F	F	T	T	T
F	T	T	F	T	F	T
T	F	F	T	T	T	T
T	F	T	T	T	F	T
T	T	F	T	T	T	T
T	T	T	T	T	F	T

- The truth table shows that the WFF $(P \rightarrow (Q \rightarrow P)) \vee \neg R$ is always true, so it satisfiable under any interpretation. But we don't need the whole truth table to prove this; the WFF is true for any interpretation in which R is false (for example).

Go to Exam Q 11

M269 2013J Exam
Soln 12

(a) *For all films, if Saira owns a copy of the film, then Saira has seen the film.*

Or more idiomatically, *Saira has seen all of the films that she owns.*

- The sentence is *FALSE*, because the enumerations of the relations show that she owns a copy of *Casablanca*, but this is not one of the films that she has seen. She also owns a copy of *El Topo*, which she has not seen either, but we only need one counter-example to show that the sentence is false.
- Soln 12 continued on next slide

Go to Exam Q 12

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Soln 12 (contd)

(b) *There exists a film, such that Jane has seen it and Jane owns it.*

Or more idiomatically, *Jane has seen at least one of the films that she owns*

- This sentence is *TRUE*. The enumerations show that she owns *Casablanca* and that she has seen it. *Django* also provides a sufficient example to show that the sentence is true.

Go to Exam Q 12

M269 2013J Exam
Soln 13

(a) This is simply the whole *operator* table.

company	*field*
Amarco	Warga
Bratape	Lolli
Rosbif	Tolstoi
Taqar	Dakhun
Bratape	Sugar

- Soln 13 continued on next slide

Go to Exam Q 13

M269 2013J Exam
Soln 13 (contd)

(b) Retaining only the rows with *production* > 2

name	*production*
Warga	3
Lolli	5
Sugar	3

- Soln 13 continued on next slide

Go to Exam Q 13

M269 2013J Exam
Soln 13

(c) Joining the tables

name	*production*	*company*
Warga	3	Amarco
Lolli	5	Bratape
Tolstoi	0.5	Rosbif
Dakhun	2	Taqar
Sugar	3	Bratape

Go to Exam Q 13

M269 2013J Exam

Soln 14

1.	$\{P \wedge (Q \wedge R)\} \vdash P \wedge (Q \wedge R)$	[Axiom schema]
2.	$\boxed{\{P \wedge (Q \wedge R)\} \vdash P}$	[1,$\wedge$-elimination left]
3.	$\varnothing \vdash (P \wedge (Q \wedge R)) \rightarrow P$	$\boxed{\text{[2,}\rightarrow\text{-introduction]}}$

▸ Go to Exam Q 14

M269 2013J Exam

Soln 15

- Only D and E are true.
- A Universal Turing Machine can compute any *computable* sequence but there are well defined problems that are not computable. (So not A)
- A lower bound may be lower than any actual algorithm. (So not B)
- Every problem in P is in NP — we just do not know if P == NP (So not C)

Go to Exam Q 15

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Soln Part2

- Part 2 Solutions

Go to Exam Q Part2

M269 2013J Exam

Soln 16

- See notes version for text

Go to Exam Q 16

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Soln 17

- See notes version for text

Go to Exam Q 17